Year 12 into 13

Summer work: A question a day

July 20th Quadratics

Write $2x^2 + 16x - 11$ in the form

$A(x + B)^2 - C$

Hence sketch the graph of

y = $2x^2 + 16x - 11$

showing all intersections with the axes and the minimum point

July 21st Surds

Simplify $\frac{8\sqrt{5}}{3-\sqrt{5}}$

July 22nd Powers

Express 8^{3x+2} in the form 2^a

July 23rd Discriminant

The equation $3x^2 + 2p = -px$ has equal roots. Find the value of p.

July 24th Disguised Quadratics

Solve $x^4 - 14x^2 + 45 = 0$

July 25th Inequalities

Find the set of values of x for which both $2x - 2 > 7 - x \; and$

$x^2 - 6x \leq 40$

July 26th Completing the Square

Complete the square for

$-0.1x^2 + 40x - 80$

July 27th Cubics

Sketch y = $9x - x^3$

July 28th Transformations

The cubic y = $x^3 - 4x^2 - 8x + 9$ is moved 2 units to the right and 3 units down. What is its new equation?

July 29th Quartics

Sketch $y = (x - 3)^2(x + 1)(2 - x)$

July 30th Reciprocal graphs

Sketch $y = \frac{15}{x^2}$

July 31st Integration

$$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}, \qquad x > 0$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

August 1st Roots

The equation $(p - 1)x^2 + 4x + (p - 5) = 0$, where p is a constant,has no real roots.

(*a*) Show that p satisfies $p^2 - 6p + 1 > 0$.

(*b*) Hence find the set of possible values of p.

August 2nd Tangents

The curve *C* has equation $y = \dfrac{(x^2 + 4)(x - 3)}{2x}$, $x \neq 0$.

(*a*) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form.

(*b*) Find an equation of the tangent to *C* at the point where $x = -1$.

Give your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

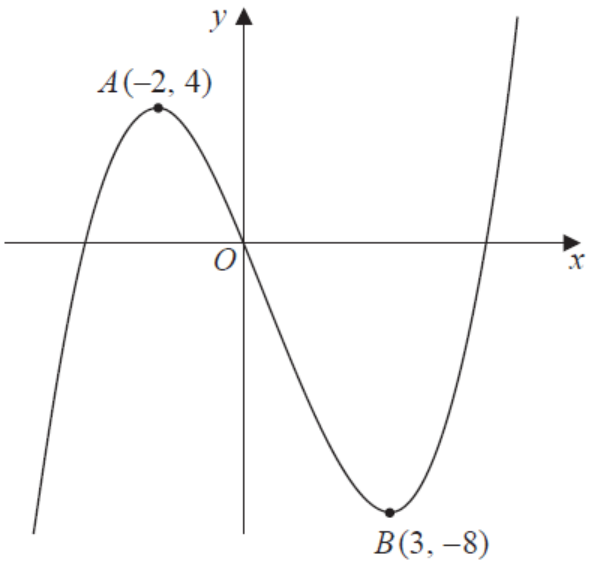
August 3rd Powers

Given that $y = 2^x$,

(*a*) express 4^x in terms of y.

(*b*) Hence, or otherwise, solve $8(4^x) - 9(2^x) + 1 = 0$.

August 4th Transformations



Sketch $y = 3f(x)$,

Sketch $y = f(x) - 4$.

August 5th Differentiation

Given that $y = 3x^2 + 6x^{\frac{1}{3}} + \dfrac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. Give each term in your answer in its simplified form.

August 6th Coordinate Geometry

The line l_1 has equation $y = -2x + 3$. The line l_2 is perpendicular to l_1 and passes through the point (5, 6).

(*a*) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers.

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(*b*) Find the x-coordinate of A and the y-coordinate of B.

Given that O is the origin,

(*c*) find the area of the triangle OAB.

August 7th Binomial Expansion

Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^{10}$, where a is a non-zero constant. Give each term in its simplest form.

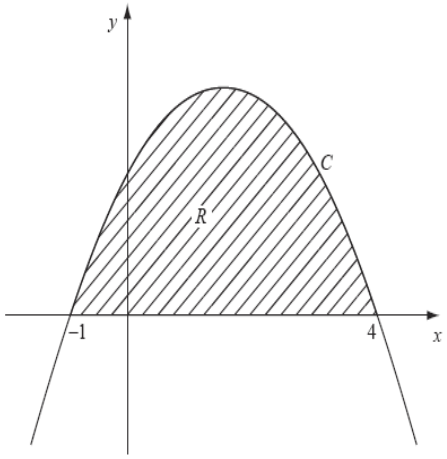
Given that, in this expansion, the coefficient of x^3 is double the coefficient of x^2,

find the value of a.

August 8th Binomial Expansion

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3 - 2x)^5$, giving each term in its simplest form.

August 9th Integration



y = (1 + x) (4 – x). Find the shaded area.

August 10th Logs

Given that $0 < x < 4$ and $\log_5 (4 - x) - 2 \log_5 x = 1$,find the value of x.

August 11th Trig

Solve, for $0 \leq x < 720°$,$4 \sin^2 x + 9 \cos x - 6 = 0$,giving your answers to 1 decimal place.

August 12th Circles

The circle , with centre A , passes through the point P with coordinates $(-9,8)$ and the point Q with coordinates $(15,-10)$.

Given that PQ is a diameter of the circle , find the coordinates for A .

August 13th Optimisation

A Solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

Show that the volume, V cm^3 , of the cylinder is given by

$V = 400r - \pi r^3$.

Given that r varies,

use calculus to find the maximum value of V, to the nearest cm^3.

(Justify that the value of V you have found is a maximum.

August 14th Travel Graphs

A car starts from rest at point *A* and moves along a straight horizontal road. The car moves with constant acceleration 1.5 m s^{-2} for the first 8 s. The car then moves with constant acceleration 0.8 m s^{-2} for the next 20 s. It then moves with constant speed for *T* seconds before slowing down with constant deceleration 2.8 m s^{-2} until it stops at a point *B*.

(*a*) Find the speed of the car 28 s after leaving *A*.

(*b*) Sketch a speed–time graph to illustrate the motion of the car as it travels from *A* to *B*.

(*c*) Find the distance travelled by the car during the first 28 s of its journey from *A*.

The distance from *A* to *B* is 2 km.

(*d*) Find the value of *T*.

August 15th Pulleys

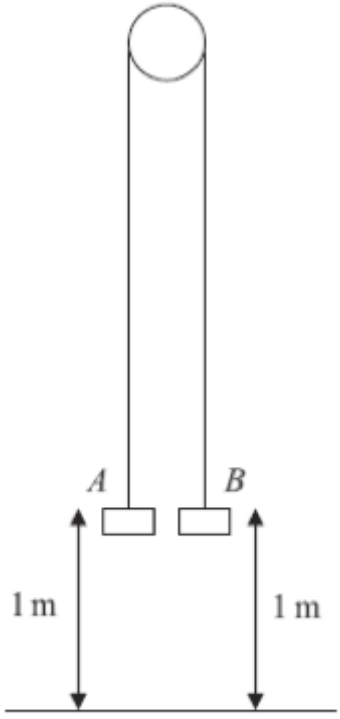


Figure 3

Two particles *A* and *B* have mass 0.4 kg and 0.3 kg respectively. The particles are attached to the ends of a light inextensible string.

The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 1 m above the floor, as shown in Figure 3.

The particles are released from rest and in the subsequent motion *B* does not reach the pulley.

When the particles have been moving for 0.5 s, the string breaks.

Given that the acceleration of *A* immediately after the particles are released is $1.4ms^{-2}$, find the further time that elapses until *B* hits the floor.

August 16th Hypothesis testing

Past records from a large supermarket show that 25% of people who buy eggs, buy organic eggs. On one particular day, a random sample of 40 people is taken from those that had bought eggs and 16 people are found to have bought organic eggs.

Test, at the 1% significance level, whether or not the proportion, , of people who bought organic eggs that day had increased. State your hypotheses clearly.

August 17th Kinematics

A car moves with constant acceleration along a straight horizontal road. The car passes the point A with speed 5 m s $^{-1}$ and 4 s later it passes the point , where $AB = 50$ m.
The acceleration of the car is 3.75 m s $^{-2}$.

When the car passes the point , it has speed 30 m s $^{-1}$.

Find the distance .

August 18th Circles

A circle has equation $x^2 + y^2 - 8x - 6y - 20 = 0$

Find the centre and radius of the circle.

August 19th Trig

Solve, for $-180^\circ \leq x < 180^\circ$, the equation $2\ tan\ x - 3\ sin\ x = 0$ giving your answers to 2 decimal places where appropriate.

August 20th Log graphs

If $y = 5x^6$, $\log_{10}$ both sides

If I graphed $\log_{10}y$ against $\log_{10}x$, give the gradient and y intercept

If $y = 3(2^x)$, $\log_{10}$ both sides

If I graphed $\log_{10}y$ against $\log_{10}x$, give the gradient and y intercept

August 21st First Principles

Differentiate $y = x^2+8x+9$ from first principles

August 22nd Binomial Distribution

$X \sim B(20, 0.61)$

Use your calculator to work out

P(X>10)

P(X<11)

P(X≤9)

P(X≥13)

P(X = 11)

August 23rd Binomial Distribution

$$X \sim B(30, 0.57)$$

Use your calculator to work out

P(X is greater than 13)

P(X is no more than 19)

P(X is at least 18)

P(X is fewer than 16)

P(X is at most 20)

$$X \sim B(10, 0.41)$$

Use your calculator to work out

P(2 < X < 4)

P($3 \leq X < 7$)

$P(4 < X \leq 9)$

$P(1 \leq X \leq 5)$

August 24th Location and Dispersion

Work out the mean and standard deviation and estimate the median and quartiles using interpolation

X	f
1-5	10
6-10	14
11-15	19

August 25th Simplifying

$$\frac{5}{\frac{2}{x}+4}$$

$$3x+11+\frac{2}{x-3}$$

$$\frac{6x}{3\left(\frac{6x}{3x+4}\right)+4}$$

$\frac{1}{2(\frac{x}{2})^2+1}$ in form $\frac{A}{4+x^2}$

August 26th Factor theorem

Factorise $6x^3 + 23x^2 - 33x + 10$ using the Factor theorem

August 27th Quadratic modelling

Find the equation of the quadratic passing through (0,0.24), (1,0.31) and (2,0.36) and complete the square for it.

August 28th Binomial twice

A fast food company has a scratchcard competition. It has ordered scratchcards for the competition and requested that 45% of the scratchcards be winning scratchcards.

A random sample of 20 of the scratchcards is collected from each of 8 of the fast food company's stores.

(a) Assuming that 45% of the scratchcards are winning scratchcards, calculate the probability that in at least 2 of the 8 stores, 12 or more of the scratchcards are winning scratchcards.

August 29th : Proof

Always, sometimes or never true:

If $ax > b$ then $x > b/a$

Please explain your answer fully.

August 30th: Proof

Always, sometimes or never true:

Two different irrational numbers multiply to get an irrational number.

Please explain your answer fully.

August 31st: Proof

Always, sometimes or never true

$X^2 + 10x + 26$ is positive

September 1st: exponentials

Solve $e^{5x-1} = 26$

September 2nd: ln

Solve ln 3 + ln 2x = 5