YEAR 12 PURE MATHS

SUMMER LEARNING PROGRAMME

Contents **Page**

1. Revision Notes 2
2. Weekly Skills Check (10 questions per week) 15
3. Mixed Topic Questions 27
4. Practice Paper B 34
5. Practice Paper C 40
6. Answers to Weekly Skills Check 47
7. Answers to Mixed Topic Questions 59
8. Answers to Practice Paper B 66
9. Answers to Practice Paper C 79

Please hand your completed and self-marked questions to your teacher in September. Worked solutions will then be made available for you to make corrections. Please then hand the questions back to your teacher and discuss areas of improvement.

AS PURE MATHS REVISION NOTES

1 SURDS

- A root such as $\sqrt{3}$ that cannot be written exactly as a fraction is IRRATIONAL
- An expression that involves irrational roots is in SURD FORM e.g. $2\sqrt{3}$
- $3 + \sqrt{2}$ and $3 - \sqrt{2}$ are CONJUGATE/COMPLEMENTARY surds – needed to rationalise the denominator

SIMPLIFYING $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ $\qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Simplify $\sqrt{75} - \sqrt{12}$

$= \sqrt{5 \times 5 \times 3} - \sqrt{2 \times 2 \times 3}$

$= 5\sqrt{3} - 2\sqrt{3}$

$= 3\sqrt{3}$

RATIONALISING THE DENOMINATOR (removing the surd in the denominator)

$a + \sqrt{b}$ and $a - \sqrt{b}$ are CONJUGATE/COMPLEMENTARY surds – the product is always a rational number

Rationalise the denominator $\frac{2}{2 - \sqrt{3}}$

$$= \frac{2}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= \frac{4 + 2\sqrt{3}}{4 + 2\sqrt{3} - 2\sqrt{3} - 3}$$

$= 4 + 2\sqrt{3}$

Multiply the numerator and denominator by the conjugate of the denominator

2 INDICES

Rules to learn

$x^a \times x^b = x^{a+b}$ $\qquad x^{-a} = \frac{1}{x^a}$ $\qquad x^0 = 1$

$x^a \div x^b = x^{a-b}$ $\qquad x^{\frac{1}{n}} = \sqrt[n]{x}$

$(x^a)^b = x^{ab}$ $\qquad x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Solve the equation

$25^x = (5^2)^x$

$$3^{2x} \times 25^x = 15$$

$$(3 \times 5)^{2x} = (15)^1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

Simplify

$(x-y)^{\frac{3}{2}}$
$= (x-y)^{\frac{1}{2}}(x-y)$

$$2x(x-y)^{\frac{3}{2}} + 3(x-y)^{\frac{1}{2}}$$

$$(x-y)^{\frac{1}{2}}(2x(x-y) + 3)$$

$$(x-y)^{\frac{1}{2}}(2x^2 - 2xy + 3)$$

3 QUADRATIC EQUATIONS AND GRAPHS

Factorising identifying the roots of the equation $ax^2 + bc + c = 0$

- Look out for the difference of 2 squares $x^2 - a^2 = (x - a)(x + a)$
- Look out for the perfect square $x^2 + 2ax + a^2 = (x + a)^2$ or $x^2 - 2ax + a^2 = (x - a)^2$
- Look out for equations which can be transformed into quadratic equations

Solve $x + 1 - \frac{12}{x} = 0$

$$x^2 + x - 12 = 0$$
$$(x + 4)(x - 3) = 0$$
$$x = 3, x = -4$$

Solve $6x^4 - 7x^2 + 2 = 0$

Let $z = x^2$

$$6z^2 - 7z + 2 = 0$$
$$(2z - 1)(3z - 2) = 0$$
$$z = \frac{1}{2} \quad z = \frac{2}{3}$$
$$x = \pm\sqrt{\frac{1}{2}} \quad x = \pm\sqrt{\frac{2}{3}}$$

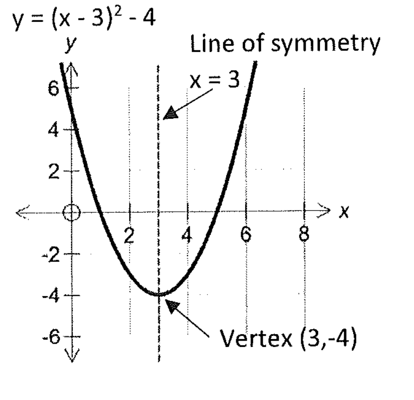
Completing the square - Identifying the vertex and line of symmetry

In completed square form

$y = (x + a)^2 - b$ the vertex is $(-a, b)$

the equation of the line of symmetry is $x = -a$

$y = (x - 3)^2 - 4$



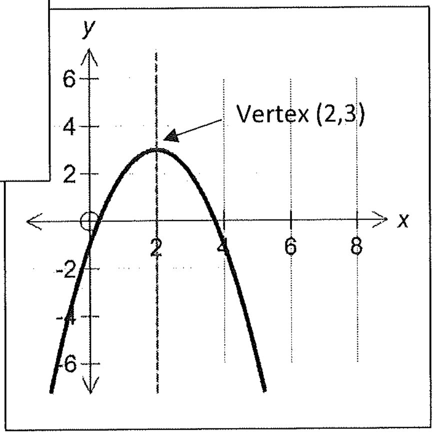
Sketch the graph of

$y = 4x - x^2 - 1$

$y = -(x^2 - 4x) - 1$

$y = -((x - 2)^2 - 4) - 1$

$y = -(x-2)^2 + 3$



Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ for solving $ax^2 + bx + c = 0$

The **DISCRIMINANT** $\mathbf{b^2 - 4ac}$ can be used to identify the number of solutions

$\mathbf{b^2 - 4ac > 0}$ there are 2 real and distinct roots (the graphs crosses the x- axis in 2 places)

$\mathbf{b^2 - 4ac = 0}$ the is a single repeated root (the x-axis is a tangent to the graph)

$\mathbf{b^2 - 4ac < 0}$ there are no 2 real roots (the graph does not touch or cross the x-axis)

4 SIMULTANEOUS EQUATIONS

Solving by elimination

$3x - 2y = 19$	$\times 3$	$9x - \mathbf{6y} = 57$
$2x - 3y = 21$	$\times 2$	$\underline{4x - \mathbf{6y} = 42}$
		$5x - 0y = 15$

$x = 3$ ($9 - 2y = 19$) $y = -5$

Solving by substitution

$x + y = 1$ rearranges to $y = 1 - x$

$x^2 + y^2 = 25$

$x^2 + (1 - x)^2 = 25$

$x^2 + 1 - 2x + x^2 - 25 = 0$

$2x^2 - 2x - 24 = 0$

$2(x^2 - x - 12) = 0$

$2(x - 4)(x + 3) = 0$ $x = 4$, $y = -3$ $x = -3$, $y = 4$

If when solving a pair of simultaneous equations, you arrive with a quadratic equation to solve, this can be used to determine the relationship between the graphs of the original equations

Using the discriminant

$b^2 - 4ac > 0$ the graphs intersect at 2 distinct points

$b^2 - 4ac = 0$ the graphs intersect at 1 point (tangent)

$b^2 - 4a < 0$ the graphs do not intersect

5 INEQUALITIES

Linear Inequality

This can be solved like a linear equation except that

Multiplying or Dividing by a negative value reverses the inequality

Solve $10 - 3x < 4$

$-3x < -6$

$x > 2$

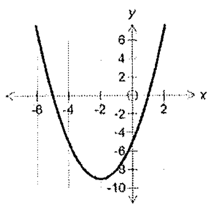
Quadratic Inequality – always a good idea to sketch the graph!

Solve $\mathbf{x^2 + 4x - 5 < 0}$

$x^2 + 4x - 5 = 0$

$(x - 1)(x + 5) = 0$

$x = 1$ $x = -5$



$x^2 + 4x - 5 < 0$

$-5 < x < 1$

which can be written as

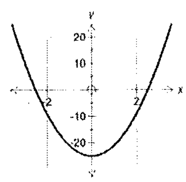
$\{x : x > -5\} \cap \{x : x < 1\}$

Solve $\mathbf{4x^2 - 25 \geq 0}$

$4x^2 - 25 = 0$

$(2x - 5)(2x + 5) = 0$

$x = \frac{5}{2}$ $x = -\frac{5}{2}$



$4x^2 - 25 \geq 0$

$x \leq -\frac{5}{2}$ or $x \geq \frac{5}{2}$

which can be written as

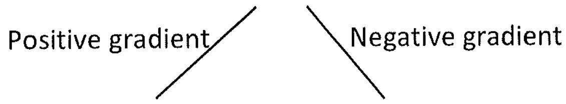
$\{x : x \leq -\frac{5}{2}\} \cup \{x : x \geq \frac{5}{2}\}$

6 GRAPHS OF LINEAR FUNCTIONS

$$y = mx + c$$

the line intercepts the y axis at (0, c)

Gradient = $\frac{change\ in\ y}{change\ in\ x}$



Finding the equation of a line with gradient m through point (a,b)

Use the equation $(y - b) = m(x - a)$

If necessary rearrange to the required form ($ax + by = c$ or $y = mx - c$)

Parallel and Perpendicular Lines

$y = m_1x + c_1$ $y = m_2x + c_2$

If $\mathbf{m_1 = m_2}$ then the lines are **PARALLEL**

If $\mathbf{m_1 \times m_2 = -1}$ then the lines are **PERPENDICULAR**

> Find the equation of the line perpendicular to the line $y - 2x = 7$ passing through point (4, -6)
>
> Gradient of $y - 2x = 7$ is 2 ($y = 2x + 7$)
> Gradient of the perpendicular line = $-\frac{1}{2}$ ($2 \times -\frac{1}{2} = -1$)
>
> Equation of the line with gradient $-\frac{1}{2}$ passing through (4, -6)
> $(y + 6) = -\frac{1}{2}(x - 4)$
> $2y + 12 = 4 - x$
> $x + 2y = -8$

Finding mid-point of the line segment joining (a,b) and (c,d)

Mid-point = $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$

Calculating the length of a line segment joining (a,b) and (c,d)

Length = $\sqrt{(c-a)^2 + (d-b)^2}$

7 CIRCLES

A circle with centre (0,0) and radius r has the equations $x^2 + y^2 = r^2$

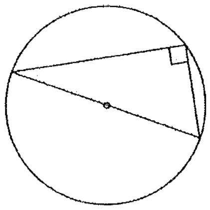
A circle with centre (a,b) and radius r is given by $(x - a)^2 + (y - b)^2 = r^2$

Finding the centre and the radius (completing the square for x and y)

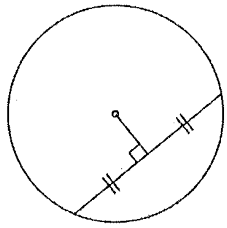
> Find the centre and radius of the circle $x^2 + y^2 + 2x - 4y - 4 = 0$
> $x^2 + 2x + y^2 - 4y - 4 = 0$
> $(x + 1)^2 - 1 + (y - 2)^2 - 4 - 4 = 0$
> $(x + 1)^2 + (y - 2)^2 = 3^2$
> Centre (-1, 2) Radius = 3

The following circle properties might be useful

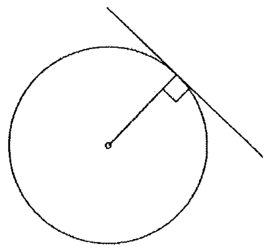
Angle in a semi-circle is a right angle



The perpendicular from the centre to a chord bisects the chord



The tangent to a circle is perpendicular to the radius



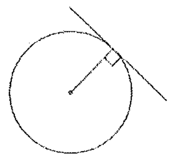
Finding the equation of a tangent to a circle at point (a,b)

The gradient of the tangent at (a,b) is perpendicular to the gradient of the radius which meets the circumference at (a, b)

Find equation of the tangent to the circle $x^2 + y^2 - 2x - 2y - 23 = 0$ at the point (5,4)

$(x - 1)^2 + (y - 1)^2 - 25 = 0$

Centre of the circle (1,1)



Gradient of radius $= \frac{4-1}{5-1} = \frac{3}{4}$ Gradient of tangent $= -\frac{4}{3}$

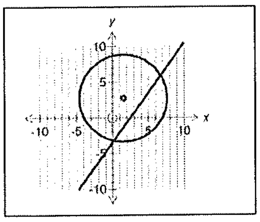
Equation of the tangent $(y - 4) = -\frac{4}{3}(x - 5)$ $3y - 12 = 20 - 4x$

$4x + 3y = 32$

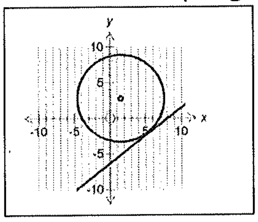
Lines and circles Solving simultaneously to investigate the relationship between a line and a circle will result in a quadratic equation.

Use the discriminant to determine the relationship between the line and the circle

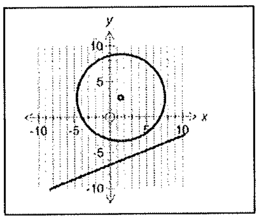
$b^2 - 4ac > 0$



$b^2 - 4ac = 0$ (tangent)



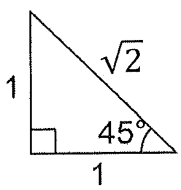
$b^2 - 4ac < 0$



8 TRIGONOMETRY

You need to learn ALL of the following

Exact Values



$\sin 45° = \frac{\sqrt{2}}{2}$

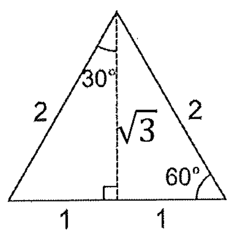
$\cos 45° = \frac{\sqrt{2}}{2}$

$\tan 45° = 1$

$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$



$\sin 60° = \frac{\sqrt{3}}{2}$

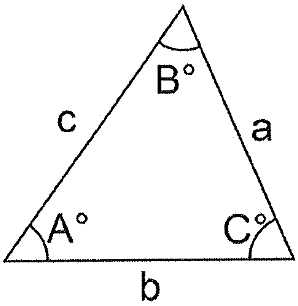
$\cos 60° = \frac{1}{2}$

$\tan 60° = \sqrt{3}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \text{ Cos } A$

Sine Rule $\frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C}$

Area of a triangle $\frac{1}{2}abSinC$

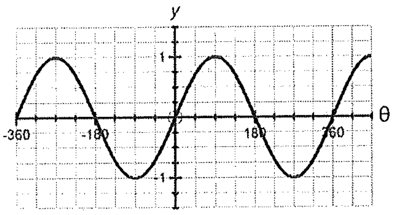


Identities

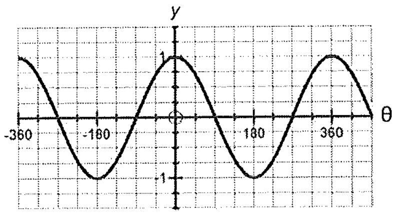
$\sin^2 x + \cos^2 x = 1$ $\tan x = \frac{\sin x}{\cos x}$

Graphs of Trigonometric Functions

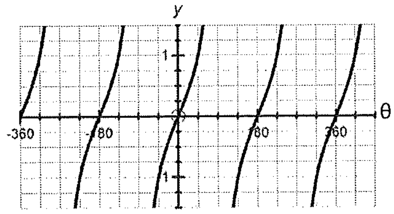
$y = \sin\theta$



$y = \cos\theta$



$y = \tan\theta$



Solve the equation $\sin^2 2\theta + \cos 2\theta + 1 = 0 \quad 0° \le \theta \le 360°$

$(1 - \cos^2 2\theta) + \cos 2\theta + 1 = 0$

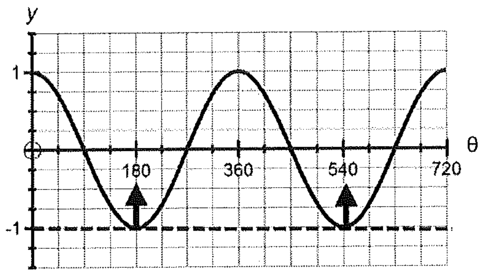
$\cos^2 2\theta - \cos 2\theta - 2 = 0$

$(\cos 2\theta - 2)(\cos 2\theta + 1) = 0$

$\cos 2\theta = 2$ (no solutions)

$\cos 2\theta = -1$ $2\theta = 180°, 540°$

$\theta = 90°, 270°$



9 **POLYNOMIALS**

- A polynomial is an expression which can be written in the form $ax^n + bx^{n-1} + cx^{n-2}$ …… when a,b, c are constants and n is a positive integer.
- The **ORDER** of the polynomial is the highest power of x in the polynomial

Algebraic Division

Polynomials can be divided to give a **Quotient** and **Remainder**

Divide $x^3 - x^2 + x + 15$ by $x + 2$

```
                x²    -3x     +7     ⟸  Quotient
x   +2 | x³    - x²    + x    + 15
         x³   + 2x²
              -3x²    +x
              -3x²    -6x
                       7x     + 15
                       7x     + 14
                               1     ⟸  Remainder
```

Factor Theorem

The factor theorem states that if (x – a) is a factor of f(x) then f(a) = 0

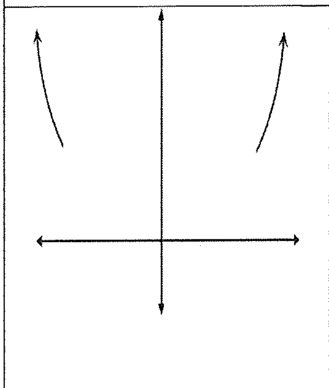
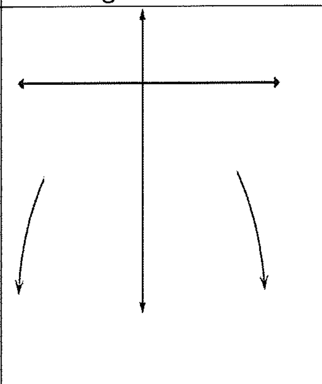
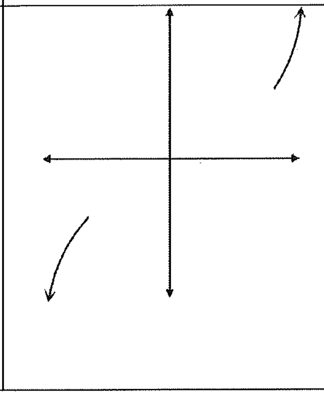
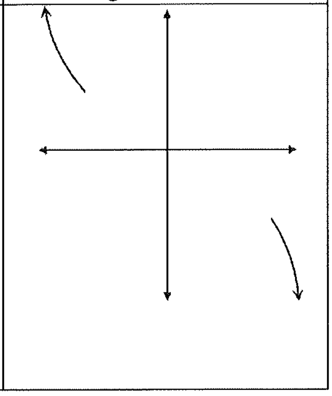
Show that (x – 3) is a factor of x^3 -19x + 30 = 0 f(x) = x^3 – 19x + 30 f(3) = 3^3 - 19×3 + 30 = 0 f(3) = 0 so (x – 3) is a factor

Sketching graphs of polynomial functions

To sketch a polynomial

- Identify where the graph crosses the y-axis (x = 0)
- Identify the where the graph crosses the x-axis, the roots of the equation y = 0
- Identify the general shape by considering the **ORDER** of the polynomial

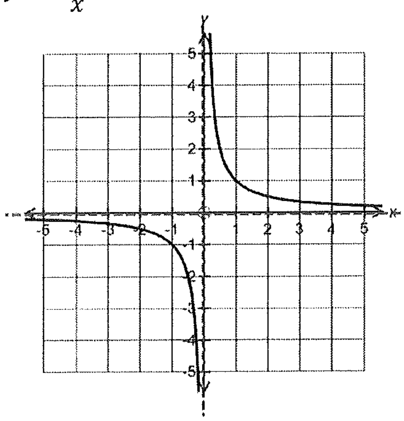
y = $ax^n + bx^{n-1} + cx^{n-2}$ … …

n is even		**n is odd**	
Positive **a > 0**	Negative **a < 0**	Positive **a > 0**	Negative **a < 0**
			

10 GRAPHS AND TRANSFORMATIONS

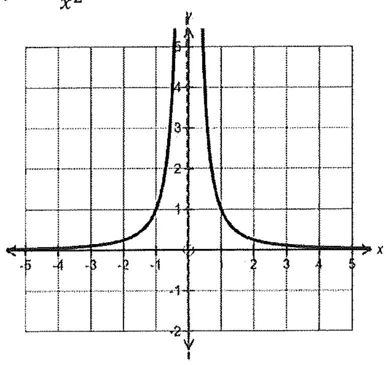
3 graphs to recognise

$y = \frac{1}{x}$



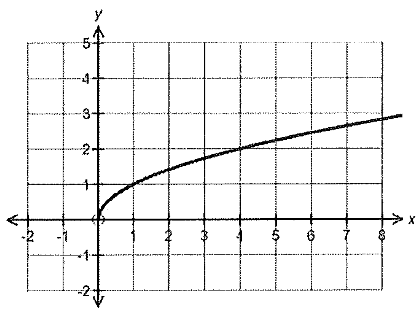
Asymptotes x= 0 and y = 0

$y = \frac{1}{x^2}$



Asymptote x = 0

$y = \sqrt{x}$



TRANSLATION

To find the equation of a graph after a translation of $\begin{bmatrix} a \\ b \end{bmatrix}$ replace x with (x - a) and replace y with (y – b)

In function notation

y = f(x) is transformed to y = f(x -a) + b

> The graph of $y = x^2 - 1$ is translated by vector $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Write down the equation of the new graph
>
> $(y + 2) = (x - 3)^2 - 1$
>
> $y = x^2 - 6x + 6$

REFLECTION

To reflect in the x-axis replace y with -y (y = -f(x))

To reflect in the y- axis replace x with -x (y = f(-x))

STRETCHING

To stretch with scale factor k in the x direction (parallel to the x-axis) replace x with $\frac{1}{k}$x   y = f($\frac{1}{k}$x)

To stretch with scale factor k in the y direction (parallel to the y-axis) replace y with $\frac{1}{k}$y   y = kf(x)

> Describe a stretch that will transform $y = x^2 + x - 1$ to the graph $y = 4x^2 + 2x - 1$
>
> $y = (2x)^2 + (2x) - 1$
>
> x has been replaced by 2x which is a **stretch of scale factor ½ parallel to the x-axis**

11 BINOMIAL EXPANSIONS

Permutations and Combinations

- The number of ways of arranging n distinct objects in a line is n! = n(n - 1)(n - 2)....3 × 2 × 1
- The number of ways of arranging a selection of r object from n is $_nP_r = \frac{n!}{(n-r)!}$
- The number of ways of picking r objects from n is $_nC_r = \frac{n!}{r!(n-r)!}$

> A committee comprising of 3 males and 3 females is to be selected from a group of 5 male and 7 female members of a club. How many different selections are possible?
>
> Female Selection $_7C_3 = \frac{7!}{3!4!}$ = 35 ways   Male Selection $_5C_3 = \frac{5!}{3!2!}$ = 10 ways
>
> Total number of different selections = 35 × 10 = 350

Expansion of $(1+x)^n$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}x^3 \ldots\ldots\ldots\ldots + nx^{n-1} + x^n$$

> Use the binomial expansion to write down the first four terms of $(1 - 2x)^8$
>
> $$(1-2x)^8 = 1 + 8\times(-2x) + \frac{8\times 7}{1\times 2}(-2x)^2 + \frac{8\times 7\times 6}{1\times 2\times 3}(-2x)^3$$
>
> $$= 1 - 16x + 112x^2 - 448x^3$$

Expansion of $(a+b)^n$

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1\times 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}a^{n-3}b^3 \ldots\ldots\ldots\ldots + nab^{n-1} + b^n$$

> Find the coefficient of the x^3 term in the expansion of $(2 + 3x)^9$
>
> $(3x)^3$ must have 2^6 as part of the coefficient (3 + 6 = 9)
>
> $\frac{9\times8\times7}{1\times2\times3}\times 2^6 \times (3x)^3$ = 145152 (x^3)

12 DIFFERENTIATION

- The gradient is denoted by $\frac{dy}{dx}$ if y is given as a function of x
- The gradient is denoted by f'(x) is the function is given as f(x)

$y = x^n \quad \frac{dy}{dx} = nx^{n-1}$ $\qquad$ $y = ax^n \quad \frac{dy}{dx} = nax^{n-1}$ $\qquad$ $y = a \quad \frac{dy}{dx} = 0$

Using Differentiation

Tangents and Normals

The gradient of a curve at a given point = gradient of the tangent to the curve at that point
The gradient of the **normal** is perpendicular to the gradient of the tangent that point

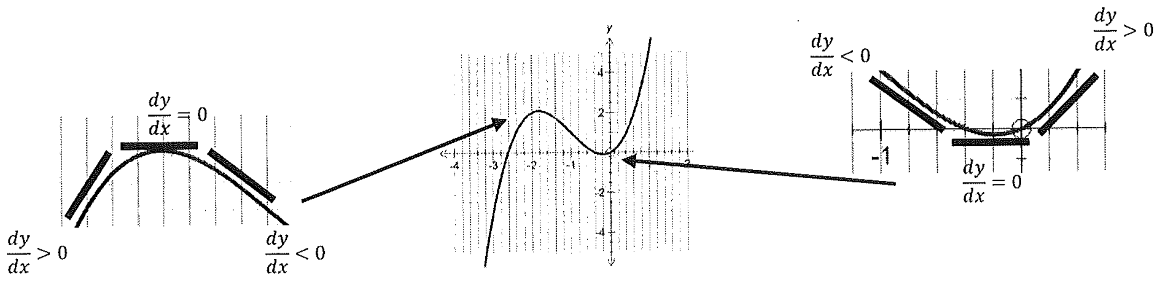
> Find the equation of the normal to the curve y = 8x – x² at the point (2,12)
>
> $\frac{dy}{dx} = 8 - 2x$ Gradient of tangent at (2,12) = 8 – 4 = 4
>
> Gradient of the normal = - ¼ $\qquad$ (y - 12) = -¼ (x – 2)
>
> 4y + x = 50

Stationary (Turning) Points

- The points where $\frac{dy}{dx} = 0$ are stationary points (turning points) of a graph
- The nature of the turning points can be found by:

Calculating the gradient close to the point

Maximum point **Minimum Point**



Differentiating (again) to find $\frac{d^2y}{dx^2}$ or f''(x)

Maximum if $\frac{d^2y}{dx^2} < 0$

Minimum if $\frac{d^2y}{dx^2} > 0$

Find and determine the nature of the turning points of the curve y = 2x³ - 3x² + 18

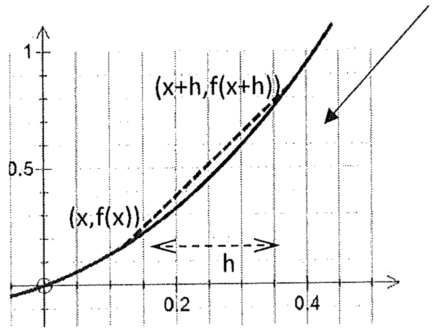
$\frac{dy}{dx} = 6x^2 - 6x$ $\quad \frac{dy}{dx} = 0$ at a turning point

6x(x - 1) = 0 Turning points at (0, 18) and (1,17)

$\frac{d^2y}{dx^2} = 12x - 6$ x = 0 $\frac{d^2y}{dx^2} < 0$ (0,18) is a maximum

x = 1 $\frac{d^2y}{dx^2} > 0$ (1,17) is a minimum

Differentiation from first principles



As h approaches zero the gradient of the chord gets closer to being the gradient of the tangent at the point

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

Find from first principles the derivative of $x^3 - 2x + 3$

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$= \lim_{h \to 0}\left(\frac{(x+h)^3 - 2(x+h) + 3 - (x^3 - 2x + 3)}{h}\right)$$

$$= \lim_{h \to 0}\left(\frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2x - 2h + 3 - x^3 + 2x - 3}{h}\right)$$

$$= \lim_{h \to 0}\left(\frac{3x^2h + 3xh^2 + h^3 - 2h}{h}\right)$$

$$= \lim_{h \to 0}(3x^2 + 3xh + h^2 - 2)$$

$$= 3x^2 - 2$$

13 INTEGRATION

Integration is the reverse of differentiation

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ (c is the constant of integration)

Given that f'(x) = 8x³ – 6x and that f(2) = 9, find f(x)

f(x) = $\int 8x^3 - 6x \, dx = 2x^4 - 3x^2 + c$

f(2) = 9 $\quad$ 2×2⁴ - 3×2² + c = 9

20 + c = 9

c = -11

f(x) = $2x^4 - 3x^2 - 11$

AREA UNDER A GRAPH

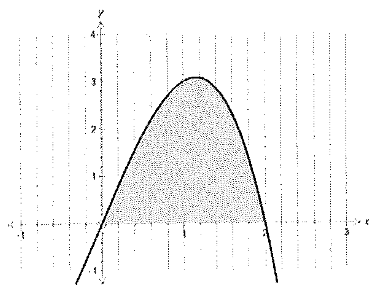
The area under the graph of y = f(x) bounded by x = a, x = b and the x-axis is found by evaluating the **definite integral** $\int_a^b f(x)dx$

Calculate the area under the graph y = 4x – x³ between x = 0 and x = 2

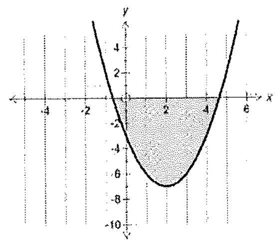
$\int_0^2 4x - x^3 dx$

$= \left[2x^2 - \frac{x^4}{4}\right]_0^2$

$= (8-4)-(0-0)$
$= 4$



An area below the x-axis has a **negative value**



14 VECTORS

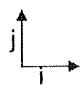
A vector has two properties **magnitude** (size) and **direction**

NOTATION

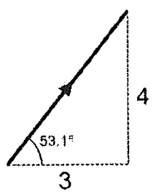
Vectors can be written as

a = $\binom{3}{4}$

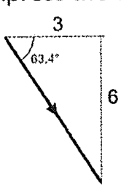
a = 3i + 4j where i and j perpendicular vectors both with magnitude 1



Magnitude-direction form (5, 53.1°) also known as **polar** form
The direction is the angle the vector makes with the **positive x axis**



Express the vector **p** = 3i – 6j in polar form



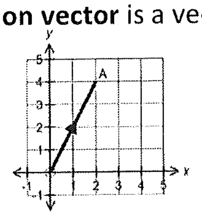
|**p**| = $\sqrt{3^2+(-6)^2}$

= $3\sqrt{5}$

p = ($3\sqrt{5}$, 63.4°)

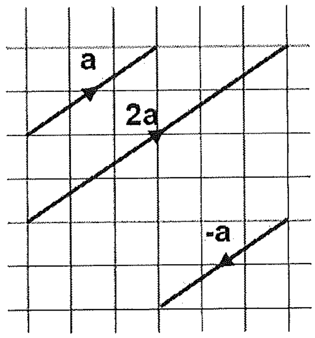
The **Magnitude** of vector **a** is denoted by |**a**| and can be found using Pythagoras |**a**| = $\sqrt{3^2+4^2}$
A **Unit Vector** is a vector which has magnitude 1

A **position vector** is a vector that starts at the origin (it has a fixed position)



$\overrightarrow{OA} = \binom{2}{4}$ $\quad 2i + 4j$

ARITHMETIC WITH VECTORS



Multiplying by a scalar (number)

$\mathbf{a} = \begin{pmatrix}3\\2\end{pmatrix}$ 3i + 2j

$2\mathbf{a} = 2\begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}6\\4\end{pmatrix}$ 6i + 4j

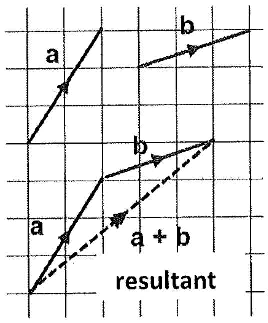
a and **2a** are **parallel** vectors

Multiplying by -1 reverses the direction of the vector

Addition of vectors

$\mathbf{a} = \begin{pmatrix}2\\3\end{pmatrix}$ $\mathbf{b} = \begin{pmatrix}3\\1\end{pmatrix}$

$\mathbf{a} + \mathbf{b} = \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}5\\4\end{pmatrix}$

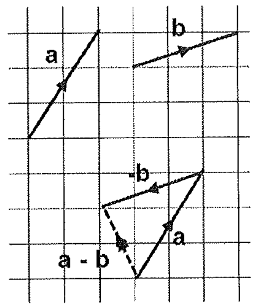


Subtraction of vectors

$\mathbf{a} = \begin{pmatrix}2\\3\end{pmatrix}$ $\mathbf{b} = \begin{pmatrix}3\\1\end{pmatrix}$

$\mathbf{a} - \mathbf{b} = \begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}-1\\2\end{pmatrix}$

This is really **a + -b**



A and B have the coordinates (1,5) and (-2,4).

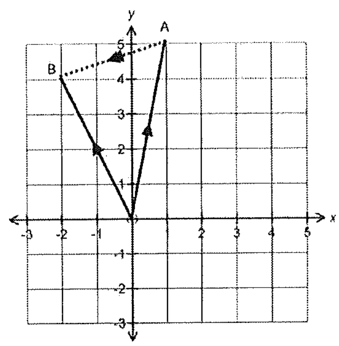
a) Write down the position vectors of A and B

$\overrightarrow{OA} = \begin{pmatrix}1\\5\end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix}-2\\4\end{pmatrix}$

b) Write down the vector of the line segment joining A to B

$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$ *or* $\overrightarrow{OB} - \overrightarrow{OA}$

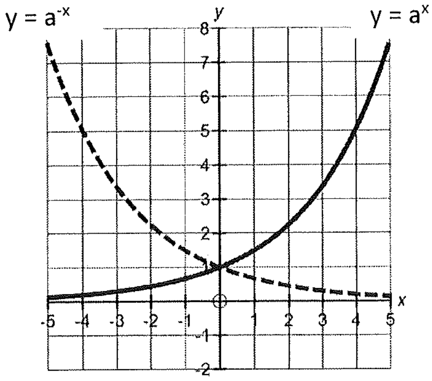
$\overrightarrow{AB} = \begin{pmatrix}-2\\4\end{pmatrix} - \begin{pmatrix}1\\5\end{pmatrix} = \begin{pmatrix}-3\\-1\end{pmatrix}$



15 LOGARITHMS AND EXPONENTIALS

- A function of the form $y = a^x$ is an exponential function
- The graph of $y = a^x$ is positive for all values of x and passes through (0,1)
- A logarithm is the inverse of an exponential function

$y = a^x$ $x = \log_a y$



Logarithms – rules to learn

$\log_a a = 1$ $\log_a 1 = 0$ $\log_a a^x = x$ $a^{\log x} = x$

$\log_a m + \log_a n = \log_a mn$ $\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$ $k\log_a m = \log_a m^k$

Write the following in the form alog 2 where a is an integer 3log 2 + 2log 4 – ½log16

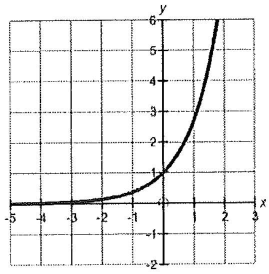
Method 1 : $\log 8 + \log 16 - \log 4 = \log\left(\frac{8\times16}{4}\right) = \log 32 = 5\log 2$

Method 2 : $3\log 2 + 4\log 2 - 2\log 2 = 5\log 2$

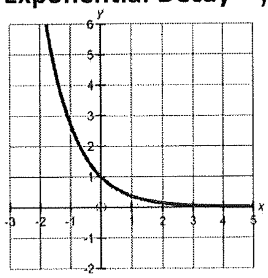
An equation of the form $a^x = b$ can be solved by taking logs of both sides

The exponential function $y = e^x$

Exponential Growth $y = e^x$



Exponential Decay $y = e^{-x}$



The inverse of $y = e^x$ is the **natural logarithm** denoted by **ln x**

Solve $2e^{x-2} = 6$ leaving your answer in exact form

$e^{x-2} = 3$

$\ln(e^{x-2}) = \ln 3$

$x - 2 = \ln 3$

$x = \ln 3 + 2$

The rate of growth/decay to find the 'rate of change' you need to differentiate to find the gradient

LEARN THIS

$y = Ae^{kx}$ $\frac{dy}{dx} = Ake^{kx}$

The number of bacteria P in a culture is modelled by $P = 600 + 5e^{0.2t}$ where t is the time in hours from the start of the experiment. Calculate the rate of growth after 5 hours

$P = 600 + 15e^{0.2t}$ $\frac{dP}{dt} = 3e^{0.2t}$

$t = 5$ $\frac{dP}{dt} = 3e^{0.2\times5}$

= 8.2 bacteria per hour

MODELLING CURVES

Exponential relationships can be changed to a linear form $y = mx + c$ allowing the constants m and c to be 'estimated' from a graph of plotted data

$y = Ax^n$ $\log y = \log (Ax^n)$ $\log y = n \log x + \log A$

$y \quad = mx \quad + \; c$

Plot log y against log x. n is the gradient of the line and log A is the y axis intercept

$y = Ab^x$ $\log y = \log (Ab^x)$ $\log y = x \log b + \log A$

$y \quad = mx \quad + \; c$

Plot log y against x. log b is the gradient of the line and log A is the y axis intercept

V and x are connected by the equation **$V = ax^b$**

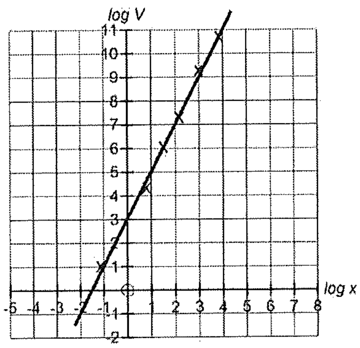
The equation is reduced to linear form by taking logs

$\log V = b \log x + \log a$

$(y = mx + c)$ (log V plotted against log x)

From the graph b = 2

$\log a = 3 \quad a = 10^3$



Gradient = 2

Intercept = 3

16 PROOF

Notation *If* $x = 3$ *then* $x^2 = 9$

$\Rightarrow$ $x = 3 \Rightarrow x^2 = 9$

$x = 3$ is a condition for $x^2 = 9$

$\Leftarrow$ $x = 3 \Leftarrow x^2 = 9$ is **not true** as x could = - 3

$\Leftrightarrow$ $x + 1 = 3 \Leftrightarrow x = 2$

Useful expressions 2n (an even number) 2n + 1 (an odd number)

Prove that the difference between the squares of any consecutive even numbers is a multiple of 4
Consecutive even numbers 2n, 2n + 2
$(2n + 2)^2 - (2n)^2$
$4n^2 + 8n + 4 - 4n^2$
$= 8n + 4$
$= 4(2n + 1)$ a multiple of 4

Find a **counter example** for the statement '2n + 4 is a multiple of 4'
n = 2 4 + 4 = 8 a multiple of 4
n = 3 6 + 4 = 10 NOT a multiple of 4

SKILLS CHECK

QUESTION 1

Solve $\frac{3}{4}(x - 3) = x - 4$

QUESTION 2

Simplify $\frac{3}{x-1} + \frac{2}{x+1}$

QUESTION 3

Express $x^2 + 6x - 10$ in the form $(x + a)^2 + b$

QUESTION 4

Simplify $\frac{x^2-x-12}{x-4}$

QUESTION 5

Solve simultaneously $x^2 + y^2 = 25 \quad x - y = 7$

WEEK 1

SKILLS CHECK

QUESTION 1

Work out

$$\int_1^2 \frac{3x - 6x^2}{x^5}\,dx$$

QUESTION 2

The points A and B have position vectors $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}-3\\11\end{bmatrix}$ respectively. M is the midpoint of the line joining A and B. Find $|\overrightarrow{BM}|$

QUESTION 3

Write the expression $\frac{1}{5}\log 32 \;-2\log 4 + \log 64$ in the form log x

QUESTION 4

Solve $3^{3x+1} = 6$ leaving your answer in exact form

QUESTION 5

Find the centre and radius of the circle given by
$x^2 + y^2 - 6x - 4y - 23 = 0$

SKILLS CHECK

QUESTION 1

Find the values of k for which the equation $8x^2 + (k+6)x + k = 0$ has a repeated root

QUESTION 2

Find the values of p for which the equation $x^2 + 2px + 1 = 0$ has no real roots

QUESTION 3

Find the equation of the line parallel to the line 6y + 3x = -4 passing through point (-3,4). Give your answer in the form ax + by = c

QUESTION 4

Use the binomial expansion to write down the first four terms of $(1 - 4x)^{10}$

QUESTION 5

Find the coordinates of the stationary points of the curve $y = 2x^3 - 24x$

WEEK 2

SKILLS CHECK

QUESTION 1

Find the coefficient of the x^4 term in the expansion of $(x-1)(1+2x)^7$

QUESTION 2

Show that $1 - \frac{sin\theta cos\theta}{tan\theta} = sin^2\theta$

QUESTION 3

If y = x(4 - x) calculate the finite area enclosed by the curve and the x - axis

QUESTION 4

If $q = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ find the vector parallel to q with magnitude 25

QUESTION 5

The graph of $y = x^2 - 2x$ is stretched by scale factor ½ parallel to the x-axis.
Find the equation of the resulting graph

WEEK 2

SKILLS CHECK

QUESTION 1

Find the values of k for which the equation $9x^2 + kx + k - 5 = 0$ has a repeated root

QUESTION 2

Find the values of p for which the equation $3x^2 + px + 3 = 0$ has real and distinct roots

QUESTION 3

Find the equation of the line through point (2,-3) which is perpendicular to the line passing through points (2, -3) and (4,5). Give your answer in the form ax + by = c

QUESTION 4

Use the binomial expansion to write down the first three terms of $(2 - 3x)^{10}$

QUESTION 5

Find the value of x $2^x \times \frac{1}{4} \times 8 = 2^7$

WEEK 3

SKILLS CHECK

QUESTION 1

Sketch the graph of y = x(x - 1)(x - 3). Calculate the total area bounded by the graph of y and the x axis between x = 0 and x = 3

QUESTION 2

Solve $3tan\theta sin\theta = cos\theta$ for $0° < \theta < 360°$

QUESTION 3

Solve ln x = ln (x + 4) - ln(x + 1)

QUESTION 4

Given that $y = 2\sqrt{x} - ax + 10$ passes through the point (1,6) find the x-coordinate of the stationary point

QUESTION 5

Find the coefficient of the x^5 term in the expansion of $\left(\frac{1}{3} - 3x\right)^{10}$

WEEK 3

SKILLS CHECK

QUESTION 1

Find the coordinates of the stationary point of $y = 2x(x^3 + 32)$

QUESTION 2

Write down a vector parallel to the vector $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$ with magnitude 20

QUESTION 3

Solve $\log_3(4x + 1) = 2$

QUESTION 4

The value of a car is depreciating. After t years it is worth (£V) is given by $V = 15000e^{-0.3t}$. After how many years will it be worth less than £5000 (3 s.f.)

.

QUESTION 5

Points A (-1,2) and B(3,5) are end points of a radius of a circle. The x-axis is a tangent to the circle. Find the equation of the circle.

SKILLS CHECK

QUESTION 1

$y = \left(x + \frac{1}{x}\right)\left(\frac{1}{x^2} - x\right)$ find $\frac{dy}{dx}$

QUESTION 2

Find $\int_1^2 6x^2 + 4x - 3\, dx$

QUESTION 3

Solve $2cos^2\theta - 3sin\theta = 0$ for $0° < \theta < 360°$

QUESTION 4

Find the value of x

$$27 \times \frac{1}{9} \times 3^{-x} = \frac{1}{81}$$

QUESTION 5

Divide $x^3 - 7x + 6$ by $x - 1$. Factorise completely and use this to sketch the graph of y $= x^3 - 7x + 6$

WEEK 4

SKILLS CHECK

QUESTION 1

A and B have position vectors $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ respectively. Calculate the angle between $\overrightarrow{AB}$ and **i**

QUESTION 2

Find the x-coordinates of the stationary points of the curve
$y = 5x^3 - 2x^2 - 3x + 10$

QUESTION 3

Sketch the graph of $y = 2x^2 - 7x$

QUESTION 4

The point (6,-10) lies on the graph of y = f(x). State the coordinates of its image when the graph is transformed to y = f(2x)

QUESTION 5

A (7,-1) and B(-1,5) are end points of a diameter of a circle. Find the points where the circle intersects the y – axis.

WEEK 5

SKILLS CHECK

QUESTION 1

Find $\int_1^9 1 + 2x + \sqrt{x}\, dx$

QUESTION 2

Find the equation of the normal to the curve y = $10\sqrt{x} - 10$ at the point where x = 4

QUESTION 3

ai + bj is a vector of magnitude $\sqrt{3}$ in the direction parallel to 3i – 3j
Find the exact values of and b.

QUESTION 4

Solve $\frac{4cos\theta - 1}{tan\theta} = 2sin\theta$ $\quad 0^0 < \theta < 360^0$

QUESTION 5

Find the coefficient of the x^6 term in the expansion of $\left(\frac{1}{2} + 2x\right)^{12}$

WEEK 5

SKILLS CHECK

QUESTION 1

Find the equation of the line perpendicular to the line 5y - 2x = 10 passing through point (-4,3). Give your answer in the form ax + by = c

QUESTION 2

Solve $tan^2 2\theta - 3tan2\theta + 2 = 0$ for $0° < \theta < 180°$

QUESTION 3

Find the coefficient of the 5th term in the expansion of $(2 - \frac{3x}{2})^8$

QUESTION 4

Find the equation of the tangent to the curve $y = 5 - 10x + x^3$ at the point when x = -1

QUESTION 5

The point (-1,4) lies on the graph of y = f(x). State the coordinates of its image when the graph is transformed to y = f(x-1) + 3

WEEK 6

SKILLS CHECK

QUESTION 1

Work out $\int_1^2 \left(3 - \frac{1}{x^2}\right)^2 dx$

QUESTION 2

A, B and C have coordinates (2,5) (6, -3) and (-1, 4). M is the midpoint of the line joining A and B . Find the vector $\overrightarrow{CM}$

QUESTION 3

Solve $2\log_2 x + \log_2 4 = 3$

QUESTION 4

The mass m of a radio active substance is given by the formula $m = m_0 e^{-kt}$ when t is in seconds and m_0 is the original mass. If the substance has a half life of 1 minute find the value of k (3 s.f.)

.

QUESTION 5

Find the equation of the tangent to the $x^2 + y^2 - 4x + 2y - 8 = 0$ at the point (0, 2)

WEEK 6

QUESTION 1

Points A and B have coordinates (2,7) and (4,15) respectively.

a) Find the gradient of the line AB

b) Find the equation of the line perpendicular to AB passing through the midpoint of AB

QUESTION 2

A curve has the equation $y = 4x^3 + 3x^2 - 7$

Find the coordinates of the stationary points

QUESTION 3

a) Factorise fully $x^3 - 3x^2 - 10x + 24$

b) Sketch the graph of $y = x^3 - 3x^2 - 10x + 24$

QUESTION 1

Find the radius and coordinates of the centre of the circle

$$x^2 + y^2 + 10y - 8x - 4 = 0$$

QUESTION 2

Solve the inequality

$$2x^2 + 5x - 3 > 0$$

QUESTION 3

Evaluate $\int_0^3 (3 - x)(3 + x)\,dx$

QUESTION 1

The equation $x^2 + kx + 36 = 0$ has 2 distinct real roots.

Find the set of values for k

QUESTION 2

Solve the simultaneous equations

$3x + y = 1$ $\qquad$ $3x + 2y^2 = 29$

QUESTION 3

A quarter circle has an area of 18π cm^2

Calculate the perimeter giving your answer in the form $a\sqrt{2}(b + \pi)$

QUESTION 1

The curve y = f(x) passes through points (2,4) and (-2, k).

Given that $\frac{dy}{dx} = 8x - 1$ find the value of k

QUESTION 2

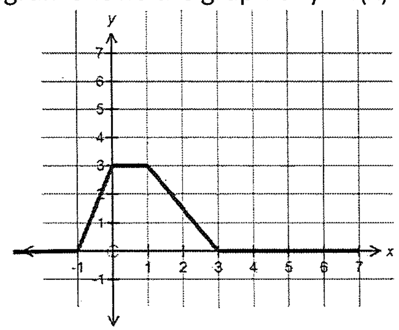
Find area enclosed by the graph of $y = x^2 + 6$ and the line y = 15

QUESTION 3

Express $\frac{(\sqrt{5}-1)^2}{\sqrt{5}+2}$ in the form $p + q\sqrt{5}$

QUESTION 1

The diagram shows the graph of y = f(x)



Sketch the graph of
y = 2f(x)

y = f(x - 2)

QUESTION 2

Find the first 4 terms in the expansion of $(1-3x)^8$

QUESTION 3

Find the equation of the normal to the curve
$y = 2\sqrt{x}$ at the point where x = 9

31

QUESTION 1

Find the equation of the normal to the curve
$y = \frac{2}{x}$ at the point where x = -2
(in the form y = mx + c)

QUESTION 2

Use the trapezium rule with 4 strips to estimate (correct to 4 decimal places)

$$\int_1^3 \frac{1}{\sqrt{2x+1}}\,dx$$

No longer in AS Pure maths.

QUESTION 3

Show that the equation

$2\log_3(x-2) - \log_3(x+2) = 2$

has 2 distinct real roots

QUESTION 1

Solve $3sin\theta + 5cos\theta = 0 \quad 0 < \theta < 360^\circ$

QUESTION 2

Solve $\log_2(x^2 + 7x + 12) - \log_2(x + 3) = 3$

QUESTION 3

Find the x^2 coefficient in the expansion of
$(1 - 4x)^3(1 + 2x)^3$

NAME:

Date to be handed in:

MARK (out of 100):

PAPER B

Qu	1	2	3	4	5	6	7	8	9	10	11	12	13

Mathematics

Advanced Subsidiary
Paper 1: Pure Mathematics

Practice Paper B:
Time 2 hours

Paper Reference
8MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Questions to revise:

1. A teacher asks one of her students to solve the equation $2\cos 2x + \sqrt{3} = 0$ for $0 \leqslant x \leqslant 180°$.

The attempt is shown below.

$2\cos 2x = -\sqrt{3}$

$\cos 2x = -\dfrac{\sqrt{3}}{2}$

$2x = \cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

$2x = 150°$

$x = 75°$

w or $x = 360° - 75° = 295°$ so reject as out of range.

(*a*) Identify the mistake made by the student.

(1)

(*b*) Write down the correct solutions to the equation.

(1)

(Total 3 marks)

---

2. Find in exact form the unit vector in the same direction as $\mathbf{a} = 4\mathbf{i} - 7\mathbf{j}$.

(Total 3 marks)

---

3. Simplify $\dfrac{6\sqrt{3}-4}{8-\sqrt{3}}$, giving your answer in the form $p\sqrt{3} - q$, where p and q are positive rational numbers.

(Total 4 marks)

---

4. (*a*) Prove that, if $1 + 3x^2 + x^3 < (1 + x)^3$, then $x > 0$.

(4)

(*b*) Show, by means of a counter example, that the inequality $1 + 3x^2 + x^3 < (1 + x)^3$ is not true for all values of x.

(2)

(Total 6 marks)

---

5. The curve with equation $y = \mathrm{h}(x)$ passes through the point (4, 19).

Given that $\mathrm{h}'(x) = 15x\sqrt{x} - \dfrac{40}{\sqrt{x}}$, find $\mathrm{h}(x)$.

(Total 6 marks)

---

6. Find all the solutions, in the interval $0 \leqslant x \leqslant 360°$, to the equation $8 - 7\cos x = 6\sin^2 x$, giving solutions to 1 decimal place where appropriate.

(Total 6 marks)

---

7. (*a*) Expand $(1 + 3x)^8$ in ascending powers of x, up to and including the term in x^3, simplifying each coefficient in the expansion.

(4)

(*b*) Showing your working clearly, use your expansion to find, to 5 significant figures, an approximation for 1.03^8.

(3)

(Total 7 marks)

---

8. (*a*) Sketch the graph $y = \log_9 (x + a)$, $a > 0$, for $x > -a$, labelling any asymptotes and points of intersection with the x-axis or y-axis. Leave your answers in terms of a where necessary.

(6)

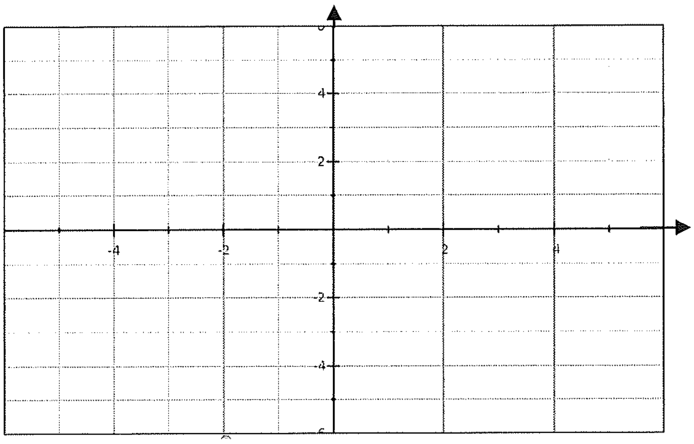
(*b*) For $x > -a$, describe, with a reason, the relationship between the graphs of $y = \log_9 (x + a)^2$ and $y = \log_9 (x + a)$.

(2)

(Total 8 marks)

---

9. (*a*) On the grid shade the region comprising all points whose coordinates satisfy the inequalities $y \leq 2x + 5$, $2y + x \leq 6$ and $y \geq 2$.



(3)

(*b*) Work out the area of the shaded region.

(5)

(Total 8 marks)

---

10. A particle P of mass 6 kg moves under the action of two forces, F_1 and F_2, where

$$F_1 = (8\mathbf{i} - 10\mathbf{j})\text{ N and } F_2 = (p\mathbf{i} + q\mathbf{j})\text{ N}, \; p \text{ and } q \text{ are constants.}$$

The acceleration of P is $\mathbf{a} = (3\mathbf{i} - 2\mathbf{j})\text{ m s}^{-2}$.

(*a*) Find, to 1 decimal place, the angle between the acceleration and **i**.

(2)

(*b*) Find the values of p and q.

(3)

(*c*) Find the magnitude of the resultant force R of the two forces F_1 and F_2. Simplify your answer fully.

(3)

(Total 8 marks)

---

11. $$f(x) = x^3 - 7x^2 - 24x + 18.$$

(*a*) Sketch the graph of the gradient function, $y = f'(x)$.

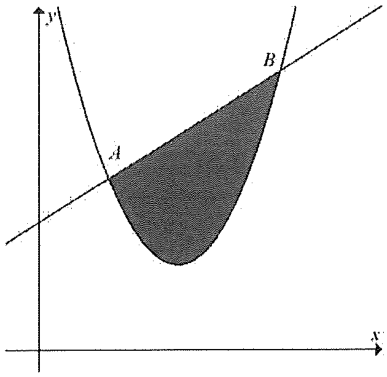
(*b*) Use algebraic methods to determine any points where the graph cuts the coordinate axes and mark these on the graph.

(*c*) Using calculus, find the coordinates of any turning points on the graph.

(Total 9 marks)

---

12 The diagram shows part of curve with equation $y = x^2 - 8x + 20$ and part of the line with equation $y = x + 6$.



(a) Using an appropriate algebraic method, find the coordinates of A and B.

(4)

The x-coordinates of A and B are denoted x_A and x_B respectively.

(b) Find the exact value of the area of the finite region bounded by the x-axis, the lines $x = x_A$ and $x = x_B$ and the line AB.

(2)

(c) Use calculus to find the exact value of the area of the finite region bounded by the x-axis, the lines $x = x_A$ and $x = x_B$ and the curve $y = x^2 - 8x + 20$.

(5)

(d) Hence, find, to one decimal place, the area of the shaded region enclosed by the curve $y = x^2 - 8x + 20$ and the line AB.

(2)

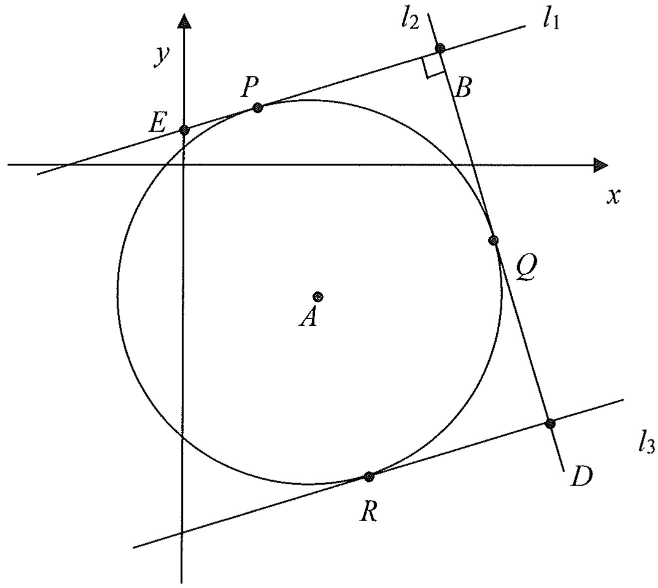
(Total 13 marks)

5

13. A is the centre of circle C, with equation $x^2 - 8x + y^2 + 10y + 1 = 0$.

P, Q and R are points on the circle and the lines l_1, l_2 and l_3 are tangents to the circle at these points respectively.

Line l_2 intersects line l_1 at B and line l_3 at D.



(a) Find the centre and radius of C.

(3)

(b) Given that the x-coordinate of Q is 10 and that the gradient of AQ is positive, find the y-coordinate of Q, explaining your solution.

(4)

(c) Find the equation of l_2, giving your answer in the form $y = mx + b$.

(4)

(d) Given that $APBQ$ is a square, find the equation of l_1 in the form $y = mx + b$.

(4)

l_1 intercepts the y-axis at E.

(e) Find the area of triangle EPA.

(4)

(Total 19 marks)

END OF PAPER (TOTAL: 100 MARKS)

NAME:

Date to be handed in:

MARK (out of 100):

PAPER C

Qu	1	2	3	4	5	6	7	8	9	10	11	12	13

Mathematics

Advanced Subsidiary
Paper 1: Pure Mathematics

Practice Paper C:
Time 2 hours

Paper Reference
8MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Questions to revise:

1. Prove, from first principles, that the derivative of $5x^3$ is $15x^2$.

(Total 4 marks)

---

2. (*a*) Sketch the graph of $y = 8^x$ stating the coordinates of any points where the graph crosses the coordinate axes.

(2)

(*b*) (i) Describe fully the transformation which transforms the graph $y = 8^x$ to the graph $y = 8^{x-1}$.

(1)

(ii) Describe the transformation which transforms the graph $y = 8^{x-1}$ to the graph $y = 8^{x-1} + 5$.

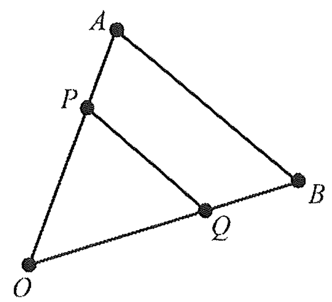
(1)

(Total 4 marks)

---

3. In $\triangle OAB$, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

P divides OA in the ratio 3 : 2 and Q divides OB in the ratio 3 : 2.



(*a*) Show that PQ is parallel to AB.

(4)

(*b*) Given that the length of AB is 10 cm, find the length of PQ.

(1)

(Total 5 marks)

---

4. $$g(x) = \frac{4}{x-6} + 5, x \in \mathbb{R}.$$

Sketch the graph $y = g(x)$.

Label any asymptotes and any points of intersection with the coordinate axes.

(Total 5 marks)

---

5. $$f(x) = 2x^3 - x^2 - 13x - 6.$$

Use the factor theorem and division to factorise f(x) completely.

(Total 6 marks)

---

6. (*a*) Fully expand $(p + q)^5$.

(2)

A fair four-sided die, numbered 1, 2, 3 and 4, is rolled 5 times.

Let p represent the probability that the number 4 is rolled on a given roll and let q represent the probability that the number 4 is not rolled on a given roll.

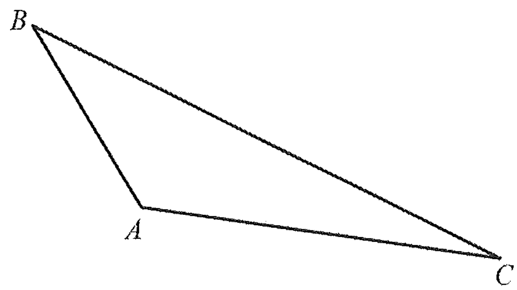
(*b*) Using the first three terms of the binomial expansion from part (a), or otherwise, find the probability that the number 4 is rolled at least 3 times.

(5)

(Total 7 marks)

---

7. In $\triangle ABC$, $\overrightarrow{AB} = -3\mathbf{i} + 6\mathbf{j}$ and $\overrightarrow{AC} = 10\mathbf{i} - 2\mathbf{j}$.



(*a*) Find the size of $\angle BAC$, in degrees, to 1 decimal place.

(5)

(*b*) Find the exact value of the area of $\triangle ABC$.

(3)

(Total 8 marks)

---

8. The points A and B have coordinates $(3k - 4, -2)$ and $(1, k + 1)$ respectively, where k is a constant.

Given that the gradient of AB is $-\frac{3}{2}$,

(*a*) show that $k = 3$,

(2)

(*b*) find an equation of the line through A and B,

(3)

(*c*) find an equation of the perpendicular bisector of A and B. Leave your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(4)

(Total 9 marks)

---

9. A stone is thrown from the top of a cliff.

The height h, in metres, of the stone above the ground level after t seconds is modelled by the function

$$h(t) = 115 + 12.25t - 4.9t^2.$$

(*a*) Give a physical interpretation of the meaning of the constant term 115 in the model.

(1)

(*b*) Write $h(t)$ in the form $A - B(t - C)^2$, where A, B and C are constants to be found.

(3)

(*c*) Using your answer to part (*b*), or otherwise, find, with justification

(i) the time taken after the stone is thrown for it to reach ground level,

(3)

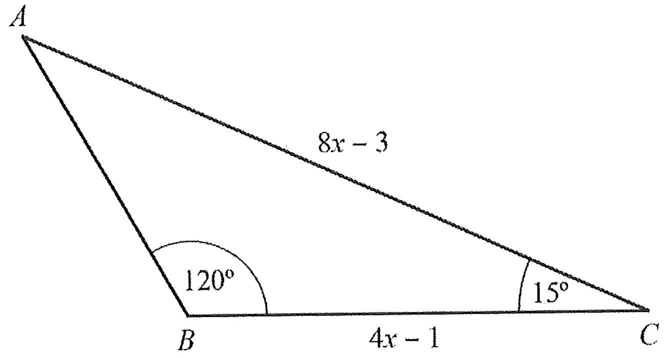
(ii) the maximum height of the stone above the ground and the time after which this maximum height is reached.

(2)

(Total 9 marks)

---

10. The diagram shows $\triangle ABC$ with $AC = 8x - 3$, $BC = 4x - 1$, $\angle ABC = 120°$ and $\angle ACB = 15°$.



(*a*) Show that the exact value of x is $\dfrac{9+\sqrt{6}}{20}$.

(7)

(*b*) Find the area of $\triangle ABC$, giving your answer to 2 decimal places.

(3)

(Total 10 marks)

---

11. (*a*) Given that $\int_a^{2a} (10 - 6x)\mathrm{d}x = 1$, find the two possible values of a.

(6)

(*b*) Labelling all axes intercepts, sketch the graph of $y = 10 - 6x$ for $0 \leq x \leq 2$.

(2)

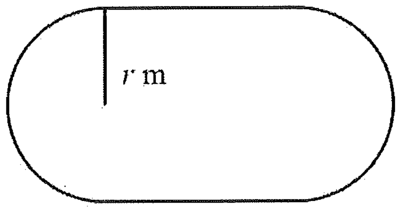
(*c*) With reference to the integral in part a and the sketch in part (*b*), explain why the larger value of a found in part (*a*) produces a solution for which the actual area under the graph between a and $2a$ is not equal to 1. State whether the area is greater than 1 or smaller than 1.

(2)

(Total 10 marks)

---

12. The diagram shows the plan of a school running track. It consists of two straight sections, which are the opposite sides of a rectangle, and two semicircular sections, each of radius r m. The length of the track is 300 m and it can be assumed to be very narrow.



(a) Show that the internal area, A m^2, is given by the formula $A = 300r - \pi r^2$.

(5)

(b) Hence find in terms of π the maximum value of the internal area.
You do not have to justify that the value is a maximum.

(6)

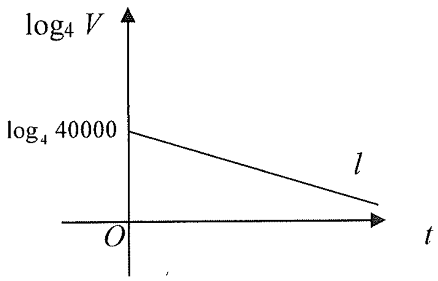
(Total 11 marks)

---

13. The value of a car, V in £, is modelled by the equation $V = ab^t$, where a and b are constants and t is the number of years since the car was purchased.

The line l shown in the diagram illustrates the linear relationship between t and $\log_4 V$ for $t \geq 0$.

The line l meets the vertical axis at $(0, \log_4 40\,000)$ as shown. The gradient of l is $-\frac{1}{10}$.



(*a*) Write down an equation for l.

(2)

(*b*) Find, in exact form, the values of a and b.

(4)

(*c*) With reference to the model, interpret the values of the constant a and b.

(2)

(*d*) Find the value of the car after 7 years.

(1)

(*e*) After how many years is the value of the car less than £10 000?

(2)

(*f*) State a limitation of the model.

(1)

(Total 12 marks)

END OF PAPER (TOTAL: 100 MARKS)

SKILLS CHECK

QUESTION 1

Solve $\frac{3}{4}(x - 3) = x - 4$

$x = 7$

QUESTION 2

Simplify $\frac{3}{x-1} + \frac{2}{x+1}$

$\frac{5x + 1}{(x-1)(x+1)}$

QUESTION 3

Express $x^2 + 6x - 10$ in the form $(x + a)^2 + b$

$(x+3)^2 - 19$

QUESTION 4

Simplify $\frac{x^2-x-12}{x-4}$

$(x+3)$

QUESTION 5

Solve simultaneously $x^2 + y^2 = 25 \quad x - y = 7$

$y = -3, \; x = 4$

$y = -4, \; x = 3$

SKILLS CHECK

QUESTION 1

Work out

$$\int_1^2 \frac{3x - 6x^2}{x^5}\,dx$$

$-1\frac{3}{8}$

QUESTION 2

The points A and B have position vectors $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}-3\\11\end{bmatrix}$ respectively. M is the midpoint of the line joining A and B. Find $|\overrightarrow{BM}|$

$\sqrt{29}$

QUESTION 3

Write the expression $\frac{1}{5}\log 32 \; - 2\log 4 + \log 64$ in the form log x

$\log 8$

QUESTION 4

Solve $3^{3x+1} = 6$ leaving your answer in exact form

$x = \frac{\log 6}{3\log 3} - \frac{1}{3}$

QUESTION 5

Find the centre and radius of the circle given by
$x^2 + y^2 - 6x - 4y - 23 = 0$

Centre (3, 2) Radius 6

WEEK 1

SKILLS CHECK

QUESTION 1

Find the values of k for which the equation $8x^2 + (k+6)x + k = 0$ has a repeated root

$k = 2 \quad k = 18$

QUESTION 2

Find the values of p for which the equation $x^2 + 2px + 1 = 0$ has no real roots

$-1 < p < 1$

QUESTION 3

Find the equation of the line parallel to the line 6y + 3x = -4 passing through point (-3,4). Give your answer in the form ax + by = c

$x + 2y = 5$

QUESTION 4

Use the binomial expansion to write down the first four terms of $(1 - 4x)^{10}$

$1 - 40x + 720x^2 - 7680x^3$

QUESTION 5

Find the coordinates of the stationary points of the curve $y = 2x^3 - 24x$

$(2, -32)$

$(-2, 32)$

WEEK 2

SKILLS CHECK

QUESTION 1

Find the coefficient of the x^4 term in the expansion of $(x-1)(1+2x)^7$

$-280x^4$

QUESTION 2

Show that $1 - \frac{sin\theta cos\theta}{tan\theta} = sin^2\theta$

QUESTION 3

If y = x(4 - x) calculate the finite area enclosed by the curve and the x - axis

$10\frac{2}{3}$

QUESTION 4

If $q = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ find the vector parallel to q with magnitude 25

$\begin{pmatrix} -15 \\ 20 \end{pmatrix}$

QUESTION 5

The graph of $y = x^2 - 2x$ is stretched by scale factor ½ parallel to the x-axis.
Find the equation of the resulting graph

$y = 4x^2 - 4x$

WEEK 2

SKILLS CHECK

QUESTION 1

Find the values of k for which the equation $9x^2 + kx + k - 5 = 0$ has a repeated root

$k = 6$ or $k = 30$

QUESTION 2

Find the values of p for which the equation $3x^2 + px + 3 = 0$ has real and distinct roots

$p < -6$ or $p > 6$

QUESTION 3

Find the equation of the line through point (2,-3) which is perpendicular to the line passing through points (2, -3) and (4,5). Give your answer in the form ax + by = c

$x + 4y = -10$

QUESTION 4

Use the binomial expansion to write down the first three terms of $(2 - 3x)^{10}$

$1024 - 15360x + 103680x^2$

QUESTION 5

Find the value of x $2^x \times \frac{1}{4} \times 8 = 2^7$

~~Grad~~ $x = 6$.

SKILLS CHECK

QUESTION 1

Sketch the graph of y = x(x - 1)(x - 3). Calculate the total area bounded by the graph of y and the x axis between x = 0 and x = 3

$\frac{37}{12}$

QUESTION 2

Solve $3tan\theta sin\theta = cos\theta$ for $0° < \theta < 360°$

$\theta = 30°, 150°, 210°, 330°$

QUESTION 3

Solve ln x = ln (x + 4) - ln(x + 1)

$x = \pm 2$

QUESTION 4

Given that $y = 2\sqrt{x} - ax + 10$ passes through the point (1,6) find the x-coordinate of the stationary point

$x = \frac{1}{36}$

QUESTION 5

Find the coefficient of the x^5 term in the expansion of $\left(\frac{1}{3} - 3x\right)^{10}$

$-252x^5$

WEEK 3

SKILLS CHECK

QUESTION 1

Find the coordinates of the stationary point of $y = 2x(x^3 + 32)$

$x = -2 \quad y = -96$

QUESTION 2

Write down a vector parallel to the vector $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$ with magnitude 20

$\begin{bmatrix} 12 \\ 16 \end{bmatrix}$

QUESTION 3

Solve $\log_3(4x + 1) = 2$

$x = 2$

QUESTION 4

The value of a car is depreciating. After t years it is worth (£V) is given by $V = 15000e^{-0.3t}$. After how many years will it be worth less than £5000 (3 s.f.)

.

$t = 3.66$ years

QUESTION 5

Points A (-1,2) and B(3,5) are end points of a radius of a circle. The x-axis is a tangent to the circle. Find the equation of the circle.

$(x-3)^2 + (y-5)^2 = 25$

WEEK 4

SKILLS CHECK

QUESTION 1

$y = \left(x + \frac{1}{x}\right)\left(\frac{1}{x^2} - x\right)$ find $\frac{dy}{dx}$

$-\frac{1}{x^2} - 2x - \frac{3}{x^4}$

QUESTION 2

Find $\int_1^2 6x^2 + 4x - 3\, dx$

17

QUESTION 3

Solve $2cos^2\theta - 3sin\theta = 0$ for $0° < \theta < 360°$

$\theta = 30°, 150°$

QUESTION 4

Find the value of x

$$27 \times \frac{1}{9} \times 3^{-x} = \frac{1}{81}$$

$x = 5$

QUESTION 5

Divide $x^3 - 7x + 6$ by $x - 1$. Factorise completely and use this to sketch the graph of $y = x^3 - 7x + 6$

$(x-1)(x-2)(x+3)$

WEEK 4

SKILLS CHECK

QUESTION 1

A and B have position vectors $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ respectively. Calculate the angle between $\overrightarrow{AB}$ and **i**

$\theta = 26.6°$

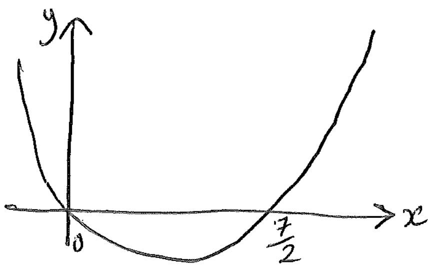
QUESTION 2

Find the x-coordinates of the stationary points of the curve
$y = 5x^3 - 2x^2 - 3x + 10$

$x = -\frac{1}{3}$ $x = \frac{3}{5}$

QUESTION 3

Sketch the graph of $y = 2x^2 - 7x$



QUESTION 4

The point (6,-10) lies on the graph of y = f(x). State the coordinates of its image when the graph is transformed to y = f(2x)

(3, -10)

QUESTION 5

A (7,-1) and B(-1,5) are end points of a diameter of a circle. Find the points where the circle intersects the y – axis.

(0, 6) (0, -2)

SKILLS CHECK

QUESTION 1

Find $\int_1^9 1 + 2x + \sqrt{x}\, dx$

$105\frac{1}{3}$

QUESTION 2

Find the equation of the normal to the curve y = $10\sqrt{x} - 10$ at the point where x = 4

$5y + 2x = 58$

QUESTION 3

ai + bj is a vector of magnitude $\sqrt{3}$ in the direction parallel to 3i – 3j
Find the exact values of and b.

$a = \sqrt{\frac{3}{2}}$ $b = -\sqrt{\frac{3}{2}}$

QUESTION 4

Solve $\frac{4cos\theta - 1}{tan\theta} = 2sin\theta$ $0^0 < \theta < 360^0$

$\theta = 48.2°, 120°, 240°, 312°$

QUESTION 5

Find the coefficient of the x^6 term in the expansion of $\left(\frac{1}{2} + 2x\right)^{12}$

$924x^6$

WEEK 5

SKILLS CHECK

QUESTION 1

Find the equation of the line perpendicular to the line 5y - 2x = 10 passing through point (-4,3). Give your answer in the form ax + by = c

$5x + 2y = -14$

QUESTION 2

Solve $tan^2 2\theta - 3tan2\theta + 2 = 0$ for $0° < \theta < 180°$

$\theta = 22.5°, 31.7°, 113°, 122°$ (3sf)

QUESTION 3

Find the coefficient of the 5th term in the expansion of $(2 - \frac{3x}{2})^8$

$5670(x^4)$

QUESTION 4

Find the equation of the tangent to the curve $y = 5 - 10x + x^3$ at the point when x = -1

$y + 7x = 7$

QUESTION 5

The point (-1,4) lies on the graph of y = f(x). State the coordinates of its image when the graph is transformed to y = f(x-1) + 3

$(0, 7)$

WEEK 6

SKILLS CHECK

QUESTION 1

Work out $\int_1^2 \left(3 - \frac{1}{x^2}\right)^2 dx$

$6\frac{7}{24}$

QUESTION 2

A, B and C have coordinates (2,5) (6, -3) and (-1, 4). M is the midpoint of the line joining A and B . Find the vector $\overrightarrow{CM}$

$\begin{bmatrix} 5 \\ -3 \end{bmatrix}$

QUESTION 3

Solve $2\log_2 x + \log_2 4 = 3$

$x = \sqrt{2}$

QUESTION 4

The mass m of a radio active substance is given by the formula $m = m_0 e^{-kt}$ when t is in seconds and m_0 is the original mass. If the substance has a half life of 1 minute find the value of k (3 s.f.)

$\frac{\ln 2}{60} = 0.0116$ (3sf)

QUESTION 5

Find the equation of the tangent to the $x^2 + y^2 - 4x + 2y - 8 = 0$ at the point (0, 2)

$3y - 2x = 6$

WEEK 6

QUESTION 1

Points A and B have coordinates (2,7) and (4,15) respectively.

a) Find the gradient of the line AB

Gradient = 4

b) Find the equation of the line perpendicular to AB passing through the midpoint of AB (in the form $ay + bx = c$)

$4y + x = 47$

QUESTION 2

A curve has the equation $y = 4x^3 + 3x^2 - 7$

Find the coordinates of the stationary points

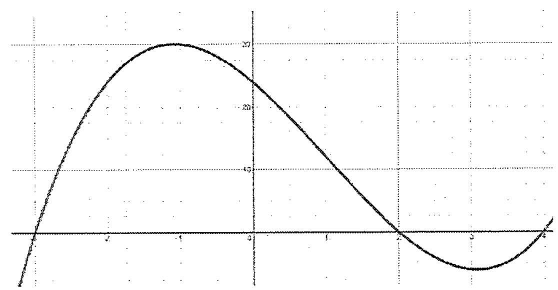
$(0, -7)$ $\left(-\frac{1}{2}, -\frac{27}{4}\right)$

QUESTION 3

a) Factorise fully $x^3 - 3x^2 - 10x + 24$

$(x - 2)(x + 3)(x - 4)$

b) Sketch the graph of $y = x^3 - 3x^2 - 10x + 24$



QUESTION 1

Find the radius and coordinates of the centre of the circle

$$x^2 + y^2 + 10y - 8x - 4 = 0$$

centre (4, -5) radius $3\sqrt{5}$

QUESTION 2

Solve the inequality

$$2x^2 + 5x - 3 > 0$$

$x < -3$, $x > \frac{1}{2}$

QUESTION 3

Evaluate $\int_0^3 (3 - x)(3 + x)dx$

= 18

60

QUESTION 1

The equation $x^2 + kx + 36 = 0$ has 2 distinct real roots.

Find the set of values for k

$k < -12$, $k > 12$

QUESTION 2

Solve the simultaneous equations

$3x + y = 1$ $\quad$ $3x + 2y^2 = 29$

$(-1, 4)$ $\quad$ $\left(\frac{3}{2}, -\frac{7}{2}\right)$

QUESTION 3

A quarter circle has an area of 18π cm^2

Calculate the perimeter giving your answer in the form $a\sqrt{2}(b + \pi)$

$3\sqrt{2}(4 + \pi)$

61

QUESTION 1

The curve $y = f(x)$ passes through points (2,4) and (-2, k).

Given that $\frac{dy}{dx} = 8x - 1$ find the value of k

$k = 8$

QUESTION 2

Find area enclosed by the graph of $y = x^2 + 6$ and the line $y = 15$

36

QUESTION 3

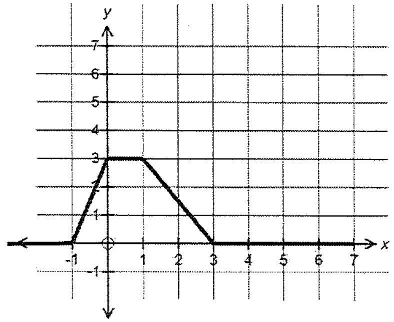
Express $\frac{(\sqrt{5}-1)^2}{\sqrt{5}+2}$ in the form $p + q\sqrt{5}$

$10\sqrt{5} - 22$

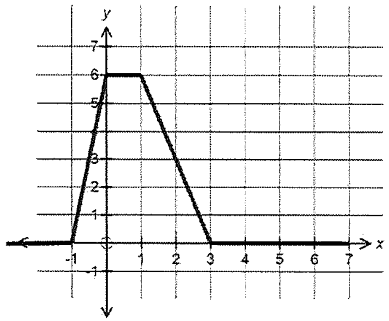
62

QUESTION 1

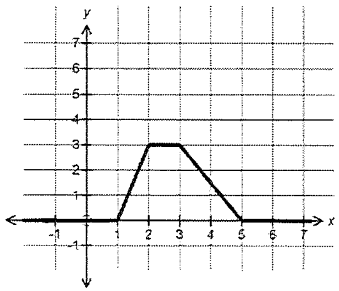
The diagram shows the graph of $y = f(x)$



Sketch the graph of
$y = 2f(x)$



$y = f(x - 2)$



QUESTION 2

Find the first 4 terms in the expansion of
$(1- 3x)^8$

$1 - 24x + 252x^2 - 1512x^3$

QUESTION 3

Find the equation of the normal to the curve
$y = 2\sqrt{x}$ at the point where $x = 9$

$y = 33 - 3x$

QUESTION 1

Find the equation of the normal to the curve $y = \frac{2}{x}$ at the point where $x = -2$ (in the form $y = mx + c$)

$y = 2x + 3$

QUESTION 2

Use the trapezium rule with 4 strips to estimate

$$\int_{1}^{3} \frac{1}{\sqrt{2x+1}}\,dx$$

0.9116

No longer in AS pure maths

QUESTION 3

Show that the equation

$$2\log_3(x-2) - \log_3(x+2) = 2$$

has 2 distinct real roots

Discriminant > 0

QUESTION 1

Solve $3sin\theta + 5cos\theta = 0 \quad 0 < \theta < 2\pi$ 360°

~~θ = 2.11, 2.25~~

θ = 121°, 301°

QUESTION 2

Use the trapezium rule with 4 strips to estimate

Solve $\log_2(x^2 + 7x + 12) - \log_2(x + 3) = 3$

X = 4

QUESTION 3

Find the x^2 coefficient in the expansion of $(1 - 4x)^3(1 + 2x)^3$

- 12

65

Advanced Subsidiary

Paper 1: Pure Mathematics

PAPER B Mark Scheme

1	Any reasonable explanation. For example, the student did not correctly find all values of $2x$ which satisfy $\cos 2x = -\frac{\sqrt{3}}{2}$. Student should have subtracted 150° from 360° first, and then divided by 2. N.B. If insufficient detail is given but location of error is correct then mark can be awarded from working in part (b).	**B1**
		(1 mark)
	$x = 75°$	**B1**
	$x = 105°$	**B1**
		(2 marks)
		Total 3 marks

NOTE: 1a: Award the mark for a different explanation that is mathematically correct, provided that the explanation is clear and not ambiguous.

2	Makes an attempt to use Pythagoras' theorem to find $\lvert\mathbf{a}\rvert$. For example, $\sqrt{(4)^2 + (-7)^2}$ seen.	**M1**
	$\sqrt{65}$	**A1**
	Displays the correct final answer. $\frac{1}{\sqrt{65}}(4\mathbf{i} - 7\mathbf{j})$	**A1**
		(3 marks)

3	Attempt to multiply the numerator and denominator by $k(8+\sqrt{3})$. For example, $\frac{6\sqrt{3}-4}{8-\sqrt{3}} \times \frac{8+\sqrt{3}}{8+\sqrt{3}}$	**M1**
	Attempt to multiply out the numerator (at least 3 terms correct). $48\sqrt{3}+18-32-4\sqrt{3}$	**M1**
	Attempt to multiply out the denominator (for example, 3 terms correct but **must** be rational or $64-3$ seen or implied). $64+8\sqrt{3}-8\sqrt{3}-3$	**M1**
	p and q stated or implied (condone if all over 61). $\frac{44}{61}\sqrt{3}-\frac{14}{61}$ or $p=\frac{44}{61}$, $q=\frac{14}{61}$	**A1**
		(4 marks)

67

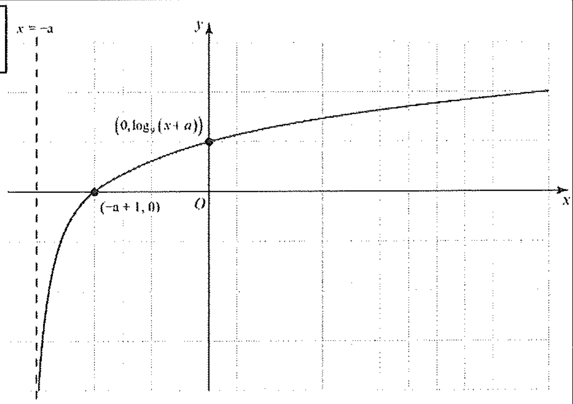
4a	Makes an attempt to expand the binomial expression $(1+x)^3$ (must be terms in x^0, x^1, x^2, x^3 and at least 2 correct).	**M1**
	$1+3x^2+x^3 < 1+3x+3x^2+x^3$	**A1**
	$0 < 3x$	**A1**
	$x > 0$* as required.	**A1***
		(4 marks)
4b	Picks a number less than or equal to zero, e.g. $x = -1$, and attempts a substitution into both sides. For example, $1+3(-1)^2+(-1)^3 < 1+3(-1)+3(-1)^2+(-1)^3$	**M1**
	Correctly deduces for their choice of x that the inequaltity does not hold. For example, $3 \nless 0$	**A1**
		(2 marks)
		Total 6 marks

5	Uses laws of indices correcty at least once anywhere in solution (e.g. $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ or $\sqrt{x} = x^{\frac{1}{2}}$ or $x\sqrt{x} = x^{\frac{3}{2}}$ seen or implied).	**B1**
	Makes an attempt at integrating $\mathrm{h}'(x) = 15x^{\frac{3}{2}} - 40x^{-\frac{1}{2}}$ Raising at least one x power by 1 would constitute an attempt.	**M1**
	Fully correct integration. $6x^{\frac{5}{2}} - 80x^{\frac{1}{2}}$ (no need for $+C$ here).	**A1**
	Makes an attenpt to substitute (4, 19) into the integrated expression. For example, $19 = 6 \times 4^{\frac{5}{2}} - 80 \times 4^{\frac{1}{2}} + C$ is seen.	**M1**
	Finds the correct value of C. $C = -13$	**A1**
	States fully correct final answer $\mathrm{h}(x) = 6x^{\frac{5}{2}} - 80\sqrt{x} - 13$ or any equivalent form.	**A1**
		(6 marks)

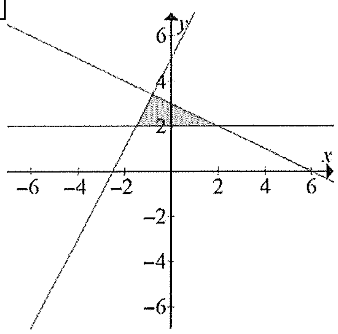
NOTES: Award all 6 marks for a fully correct final answer, even if some working is missing.

6	States $\sin^2 x + \cos^2 x = 1$ or implies this by making a substitution. $8 - 7\cos x = 6(1 - \cos^2 x)$	**M1**
	Simplifies the equation to form a quadratic in $\cos x$. $6\cos^2 x - 7\cos x + 2 = 0$	**M1**
	Correctly factorises this equation. $(3\cos x - 2)(2\cos x - 1) = 0$ or uses equivalent method for solving quadratic (can be implied by correct solutions).	**M1**
	Correct solution. $\cos x = \frac{2}{3}$ or $\frac{1}{2}$	**A1**
	Finds one correct solution for x. (48.2°,60°, 311.8° or 300°).	**A1**
	Finds all other solutions to the equation.	**A1**
		(6 marks)

7a	States or implies the expansion of a binomial expression to the 8th power, up to and including the x^3 term. $(a+b)^8 = {}^8C_0a^8 + {}^8C_1a^7b + {}^8C_2a^6b^2 + {}^8C_3a^5b^3 + \ldots$ or $(a+b)^8 = a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + \ldots$	**M1**
	Correctly substitutes 1 and $3x$ into the formula: $(1+3x)^8 = 1^8 + 8\times1^7\times3x + 28\times1^6\times(3x)^2 + 56\times1^5\times(3x)^3 + \ldots$	**M1**
	Makes an attempt to simplify the expression (2 correct coefficients (other than 1) or both $9x^2$ and $27x^3$). $(1+3x)^8 = 1^8 + 24x + 28\times9x^2 + 56\times27x^3 + \ldots$	**M1 dep**
	States a fully correct answer: $(1+3x)^8 = 1 + 24x + 252x^2 + 1512x^3 + \ldots$	**A1**
		(4 marks)
7b	States $x = 0.01$ or implies this by attempting the substitution: $1 + 24(0.01) + 252(0.01)^2 + 1512(0.01)^3 + \ldots$	**M1**
	Attempts to simplify this expression (2 calculated terms correct): 1 + 0.24 + 0.0252 + 0.001512	**M1**
	1.266712 = 1.2667 (5 s.f.)	**A1**
		(3 marks)
		Total 7 marks

8a 	Attempt to find intersection with x-axis. For example, $\log_9(x+a)=0$	**M1**
	Solving $\log_9(x+a)=0$ to find $x=-a+1$, so coordinates of x-intercept are $(-a+1, 0)$ oe	**A1**
	Substituting $x=0$ to derive $y=\log_9(x+a)$, so coordinates of y-intercept are $(0, \log_9(x+a))$	**B1**
	Asymptote shown at $x=-a$ stated or shown on graph.	**B1**
	Increasing log graph shown with asymptotic behaviour and single x-intercept.	**M1**
	Fully correct graph with correct asymptote, all points labelled and correct shape.	**A1**
		(6 marks)
8b $\log_9(x+a)^2 = 2\log_9(x+a)$ seen.		**M1**
The graph of $y=\log_9(x+a)^2$ is a stretch, parallel to the y-axis, scale factor 2, of the graph of $y=\log_9(x+a)$.		**A1**
		(2 marks)
		Total 8 marks

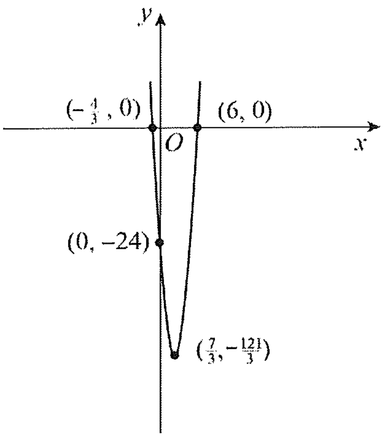
NOTES: 8a: Award all 5 points for a fully correct graph with asymptote and all points labelled, even if all working is not present

9a		Graph of $y = 2x + 5$ drawn.	**B1**
		Graph of $2y + x = 6$ drawn.	**B1**
		Graph of $y = 2$ drawn onto the coordinate grid and the triangle correctly shaded.	**B1**
			(3 marks)
9b	Attempt to solve $y = 2x + 5$ and $2y + x = 6$ simultaneously for y.		**M1**
	$y = 3.4$		**A1**
	Base of triangle = 3.5		**B1**
	Area of triangle = $\frac{1}{2} \times (\text{“}3.4\text{”} - 2) \times 3.5$		**M1**
	Area of triangle is 2.45 (units^2).		**A1**
			(5 marks)
			Total 8 marks

NOTES: 9b: It is possible to find the area of triangle by realising that the two diagonal lines are perpendicular and therefore finding the length of each line using Pythagoras' theorem. Award full marks for a correct final answer using this method.

In this case award the second and third accuracy marks for finding the lengths $\sqrt{2.45}$ and $\sqrt{9.8}$

10a	States that $\tan\theta = \pm\frac{2}{3}$ or $\theta = \tan^{-1} \pm \frac{2}{3}$ (if θ shown on diagram sign must be consistent with this).	**M1**
	Finds −33.7° (must be negative).	**A1**
		(2 marks)
10b	Makes an attempt to use the formula $\mathbf{F} = m\mathbf{a}$	**M1**
	Finds $p = 10$ Note: $8 + p = 6\times 3 \Rightarrow p = 10$	**A1**
	Finds $q = -2$ Note: $-10 + q = 6\times -2 \Rightarrow q = -2$	**A1**
		(3 marks)
10c	Attempt to find **R** (either $6(3\mathbf{i} - 2\mathbf{j})$ or $8\mathbf{i} - 10\mathbf{j} + '10'\mathbf{i} + '-2'\mathbf{j}$).	**M1**
	Makes an attempt to find the magnitude of their resultant force. For example, $\lvert R\rvert = \sqrt{'18'^2 + '12'^2}\ \left(= \sqrt{468}\right)$	**M1**
	Presents a fully simplified exact final answer. $\lvert R\rvert = 6\sqrt{13}$	**A1**
		(3 marks)
		Total 8 marks

11		
Attempts to differentiate.		M1
$f'(x) = 3x^2 - 14x - 24$		A1
States or implies that the graph of the gradient function will cut the x-axis when $f'(x) = 0$ $f'(x) = 0 \Rightarrow 3x^2 - 14x - 24 = 0$		M1
Factorises $f'(x)$ to obtain $(3x+4)(x-6) = 0$ $x = -\frac{4}{3}, x = 6$		A1
States or implies that the graph of the gradient function will cut the y-axis at $f'(0)$. Substitutes $x = 0$ into $f'(x)$ Gradient function will cut the y-axis at $(0, -24)$.		M1
Attempts to find the turning point of $f'(x)$ by differentiating (i.e. finding $f''(x)$)		M1
$f''(x) = 0 \Rightarrow 6x - 14 = 0 \Rightarrow x = \frac{7}{3}$		A1
Substitutes $x = \frac{7}{3}$ into $f'(x)$ to obtain $y = -\frac{121}{3}$		A1ft
	A parabola with correct orientation with required points correctly labelled.	A1ft
		(9 marks)

NOTES: A mistake in the earlier part of the question should not count against the students for the last part. If a student sketches a parabola with the correct orientation correctly labelled for their values, award the final mark.

Note that a fully correct sketch without all the working but with all points clearly labelled implies 8 marks in this question.

12a	Equates the curve and the line. $x^2 - 8x + 20 = x + 6$	**M1**
	Simplifies and factorises. $(x - 7)(x - 2) = 0$ (or uses other valid method for solving a quadratic equation).	**M1**
	Finds the correct coordinates of A. $A(2, 8)$.	**A1**
	Finds the correct coordinates of B. $B(7, 13)$.	**A1**
		(4 marks)
12b	Makes an attempt to find the area of the trapezium bounded by $x = 2$, $x = 7$, the x-axis and the line. For example, $\frac{5}{2}(8+13)$ or $\int_2^7 (x+6)\mathrm{d}x$ seen.	**M1**
	Correct answer. Area = 52.5 o.e.	**A1**
		(2 marks)
12c	$\int_2^7 (x^2 - 8x + 20)\mathrm{d}x$.	**B1**
	Makes an attempt to find the integral. Raising at least one x power by 1 would constitute an attempt.	**M1**
	Correctly finds $\left[\frac{1}{3}x^3 - 4x^2 + 20x\right]_2^7$	**A1**
	Makes an attempt to substitute limits into the definite integral. $\left[\left(\frac{343}{3} - 196 + 140\right) - \left(\frac{8}{3} - 16 + 40\right)\right]$	**M1**
	Correct answer seen. $\frac{95}{3}$ or $31.\dot{6}$ oe seen.	**A1**
		(5 marks)
12d	Understands the need to subtract the two areas. $\pm(52.5 - 31.\dot{6})$	**M1**
	20.8 units2 seen (must be positive).	**A1**
		(2 marks)
		Total 13 marks

NOTES: 12a: If A0A0, award A1 for full solution of quadratic equation (i.e. $x = 2$, $x = 7$).

13a	Student completes the square twice. Condone sign errors. $(x-4)^2-16+(y+5)^2-25+1=0$ $(x-4)^2+(y+5)^2=40$	**M1**
	So centre is (4, −5)	**A1**
	and radius is $\sqrt{40}$	**A1**
		(3 marks)
13b	Substitutes $x=10$ into equation (in either form). $10^2-8\times10+y^2+10y+1=0$ or $(10-4)^2+(y+5)^2=40$	**M1**
	Rearranges to 3 term quadratic in y $y^2+10y+21=0$ (could be in completed square form $(y+5)^2=4$)	**M1**
	Obtains solutions $y=-3$, $y=-7$ (must give both).	**A1**
	Rejects $y=-7$ giving suitable reason (e.g. $-7<-5$) or 'it would be below the centre' or 'AQ must slope upwards' o.e.	**B1**
		(4 marks)
13c	$m_{AQ}=\dfrac{-3-(-5)}{10-4}=\dfrac{1}{3}$	**B1**
	$m_{l_2}=-3$ (i.e. −1 over their m_{AQ})	**B1ft**
	Substitutes their Q into a correct equation of a line. For example, $-3=(-3)(10)+b$ or $y+3=-3(x-10)$	**M1**
	$y=-3x+27$	**A1**
		(4 marks)

13d	$\overrightarrow{AQ} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ o.e. (could just be in coordinate form).	**M1**
	$\overrightarrow{AP} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$ o.e. so student concludes that point P has coordinates (2, 1).	**M1**
	Substitutes their P and their gradient $\frac{1}{3}$ (m_{AQ} from 5c) into a correct equation of a line. For example, $1 = \left(\frac{1}{3}\right)(2) + b$ or $y - 1 = \left(\frac{1}{3}\right)(x - 2)$	**M1**
	$y = \frac{1}{3}x + \frac{1}{3}$	**A1**
		(4 marks)
13e	$PA = \sqrt{40}$	**B1**
	Uses Pythagoras' theorem to find $EP = \sqrt{\frac{40}{9}}$.	**B1**
	Area of EPA = $\frac{1}{2} \times \sqrt{40} \times \sqrt{\frac{40}{9}}$ (could be in two parts).	**M1**
	Area = $\frac{20}{3}$	**A1**
		(4 marks)
		Total 19 marks

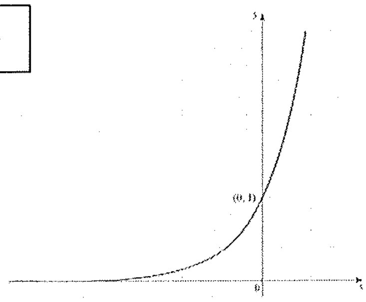
Advanced Subsidiary Paper 1: Pure Mathematics	PAPER C Mark Scheme

1	States or implies the formula for differentiation from first principles. $f(x) = 5x^3$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	**B1**
	Correctly applies the formula to the specific formula and expands and simplifies the formula. $f'(x) = \lim_{h \to 0} \frac{5(x+h)^3 - 5x^3}{h}$ $f'(x) = \lim_{h \to 0} \frac{5(x^3 + 3x^2h + 3xh^2 + h^3) - 5x^3}{h}$ $f'(x) = \lim_{h \to 0} \frac{15x^2h + 15xh^2 + 5h^3}{h}$	**M1**
	Factorises the 'h' out of the numerator and then divides by h to simplify. $f'(x) = \lim_{h \to 0} \frac{h(15x^2 + 15xh + 5h^2)}{h}$ $f'(x) = \lim_{h \to 0} (15x^2 + 15xh + 5h^2)$	**A1**
	States that as $h \to 0$, $15x^2 + 15xh + 5h^2 \to 15x^2$ o.e. so derivative $= 15x^2$ *	**A1***
		(4 marks)

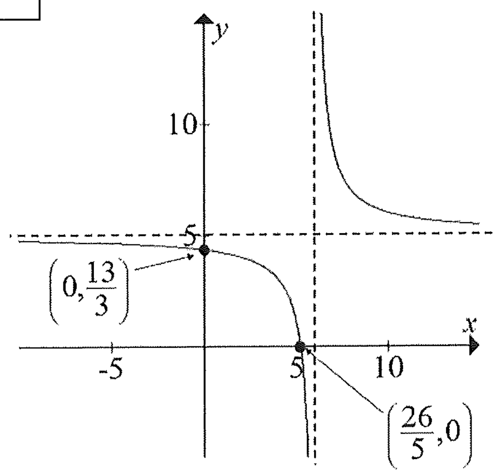
NOTES: Use of δx also acceptable.

Students must show a complete proof (without wrong working) to achieve all 4 marks.

Not all steps need to be present, and additional steps are also acceptable.

2		Graph has correct shape and does not touch x-axis.	**M1**
		The point (0, 1) is given or labelled.	**A1**
			(2 marks)
	Translation 1 unit right (or positive x direction) or by $\begin{pmatrix}1\\0\end{pmatrix}$		**B1**
	Translation 5 units up (or positive y direction) or by $\begin{pmatrix}0\\5\end{pmatrix}$		**B1**
			(2 marks)
			Total 4 marks

3a	States that $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$	**M1**
	States $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$ or $\overrightarrow{PQ} = -\frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$	**M1**
	States $\overrightarrow{PQ} = \frac{3}{5}(-\mathbf{a} + \mathbf{b})$ or $\overrightarrow{PQ} = \frac{3}{5}\overrightarrow{AB}$	**A1**
	Draws the conclusion that as $\overrightarrow{PQ}$ is a multiple of $\overrightarrow{AB}$ the two lines PQ and AB must be parallel.	**A1**
		(4 marks)
3b	$PQ = \frac{3}{5} \times 10$ cm = 6 cm cao	**B1**
		(1 mark)
		Total 5 marks

4		
	Asymptote drawn at $x = 6$	**B1**
	Asymptote drawn at $y = 5$	**B1**
	Point $\left(0, \frac{13}{3}\right)$ labelled. Condone $\frac{13}{3}$ clearly on y axis.	**B1**
	Point $\left(\frac{26}{5}, 0\right)$ labelled. Condone $\frac{26}{5}$ clearly on x axis.	**B1**
	Correctly shaped graph drawn in the correct quadrants formed by the asymptotes.	**B1**
		(5 marks)

5		
	Correctly shows that either $f(3) = 0$, $f(-2) = 0$ or $f\left(-\frac{1}{2}\right) = 0$	**M1**
	Draws the conclusion that $(x - 3)$, $(x + 2)$ or $(2x + 1)$ must therefore be a factor.	**M1**
	Either makes an attempt at long division by setting up the long division, or makes an attempt to find the remaining factors by matching coefficients. For example, stating $(x-3)(ax^2+bx+c)=2x^3-x^2-13x-6$ or $(x+2)(rx^2+px+q)=2x^3-x^2-13x-6$ or $(2x+1)(ux^2+vx+w)=2x^3-x^2-13x-6$	**M1**
	For the long division, correctly finds the the first two coefficients. For the matching coefficients method, correctly deduces that $a = 2$ and $c = 2$ or correctly deduces that $r = 2$ and $q = -3$ or correctly deduces that $u = 1$ and $w = -6$	**A1**
	For the long division, correctly completes all steps in the division. For the matching coefficients method, correctly deduces that $b = 5$ or correctly deduces that $p = -5$ or correctly deduces that $v = -1$	**A1**
	States a fully correct, fully factorised final answer: $(x - 3)(2x + 1)(x + 2)$	**A1**
		(6 marks)

NOTES: Other algebraic methods can be used to factorise h(x).

For example, if $(x - 3)$ is known to be a factor then

$2x^3 - x^2 - 13x - 6 = 2x^2(x-3) + 5x(x-3) + 2(x-3)$ by balancing (M1)

$= (2x^2 + 5x + 2)(x - 3)$ by factorising (M1)

$= (2x + 1)(x + 2)(x - 3)$ by factorising (A1)

6a	Attempt is made at expanding $(p+q)^5$. Accept seeing the coefficients 1, 5, 10, 10, 5, 1 or seeing $(p+q)^5 = {}^5C_0 p^5 + {}^5C_1 p^4 q + {}^5C_2 p^3 q^2 + {}^5C_3 p^2 q^3 + {}^5C_4 p q^4 + {}^5C_5 q^5$ o.e.	**M1**
	Fully correct answer is stated: $(p+q)^5 = p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$	**A1**
		(2 marks)
6b	States that p, or the probability of rolling a 4, is $\frac{1}{4}$	**B1**
	States that q, or the probability of not rolling a 4, is $\frac{3}{4}$	**B1**
	States or implies that the sum of the first 3 terms (or 1 − the sum of the last 3 terms) is the required probability. For example, $p^5 + 5p^4q + 10p^3q^2$ or $1 - (10p^2q^3 + 5pq^4 + q^5)$	**M1**
	$\left(\frac{1}{4}\right)^5 + 5\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right) + 10\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$ or $\frac{1}{1024} + \frac{15}{1024} + \frac{90}{1024}$ or $1 - \left(10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 + 5\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + \left(\frac{3}{4}\right)^5\right)$ or $1 - \left(\frac{270}{1024} + \frac{405}{1024} + \frac{243}{1024}\right)$	**M1**
	Either $\frac{53}{512}$ o.e. or awrt 0.104	**A1**
		(5 marks)
		Total 7 marks

7a	States or implies that $\overrightarrow{BC} = 13\mathbf{i} - 8\mathbf{j}$ o.e.	**M1**
	Recognises that the cosine rule is needed to solve for $\angle BAC$ by stating $a^2 = b^2 + c^2 - 2bc \times \cos A$	**M1**
	Makes correct substitutions into the cosine rule. $\left(\sqrt{233}\right)^2 = \left(\sqrt{45}\right)^2 + \left(\sqrt{104}\right)^2 - 2\left(\sqrt{45}\right)\left(\sqrt{104}\right) \times \cos A$ o.e.	**M1**
	$\cos A = -\frac{7}{\sqrt{130}}$ or awrt -0.614 (seen or implied by correct answer).	**M1**
	$A = 127.9°$ cao	**A1**
		(5 marks)
7b	States formula for the area of a triangle. Area $= \frac{1}{2}ab\sin C$	**M1**
	Makes correct substitutions using their values from above. Area $= \frac{1}{2}\left(\sqrt{45}\right)\left(\sqrt{104}\right)\sin 127.9...°$	**M1ft**
	Area $= 27$ (units^2)	**A1ft**
		(3 marks)
		Total 8 marks

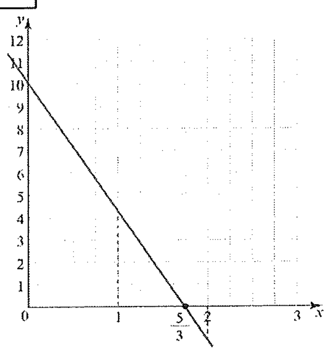
8a	Use of the gradient formula to begin attempt to find k. $\frac{k+1-(-2)}{1-(3k-4)}=-\frac{3}{2}$ or $\frac{-2-(k+1)}{3k-4-1}=-\frac{3}{2}$ (i.e. correct substitution into gradient formula and equating to $-\frac{3}{2}$).	**M1**
	$2k+6=-15+9k$ $21=7k$ $k=3$* (must show sufficient, convincing and correct working).	**A1***
		(2 marks)
8b	Student identifies the coordinates of either A or B. Can be seen or implied, for example, in the subsequent step when student attempts to find the equation of the line. $A(5, -2)$ or $B(1, 4)$.	**B1**
	Correct substitution of their coordinates into $y = mx + b$ or $y - y_1 = m(x - x_1)$ o.e. to find the equation of the line. For example, $-2=\left(-\frac{3}{2}\right)(5)+b$ or $y+2=\left(-\frac{3}{2}\right)(x-5)$ or $4=\left(-\frac{3}{2}\right)(1)+b$ or $y-4=\left(-\frac{3}{2}\right)(x-1)$	**M1**
	$y=-\frac{3}{2}x+\frac{11}{2}$ or $3x+2y-11=0$	**A1**
		(3 marks)
8c	Midpoint of AB is (3, 1) seen or implied.	**B1**
	Slope of line perpendicular to AB is $\frac{2}{3}$, seen or implied.	**B1**
	Attempt to find the equation of the line (i.e. substituting their midpoint and gradient into a correct equation). For example, $1=\left(\frac{2}{3}\right)(3)+b$ or $y-1=\frac{2}{3}(x-3)$	**M1**
	$2x-3y-3=0$ or $3y-2x+3=0$. Also accept any multiple of $2x-3y-3=0$ providing a, b and c are still integers.	**A1**
		(4 marks)
		Total **9 marks**

9a	115 (m) is the height of the cliff (as this is the height of the ball when $t = 0$). Accept answer that states 115 (m) is the height of the cliff plus the height of the person who is ready to throw the stone or similar sensible comment.	**B1**
		(1 mark)
9b	Attempt to factorise the – 4.9 out of the first two (or all) terms. $h(t) = -4.9(t^2 - 2.5t) + 115$ or $h(t) = -4.9\left(t^2 - \frac{5}{2}t\right) + 115$	**M1**
	$h(t) = -4.9(t-1.25)^2 - (-4.9)(1.25)^2 + 115$ or $h(t) = -4.9\left(t - \frac{5}{4}\right)^2 - (-4.9)\left(\frac{5}{4}\right)^2 + 115$	**M1**
	$h(t) = 122.65625 - 4.9(t-1.25)^2$ o.e. (N.B. $122.65625 = \frac{3925}{32}$) Accept the first term written to 1, 2, 3 or 4 d.p. or the full answer as shown.	**A1**
		(3 marks)
9ci	Statement that the stone will reach ground level when $h(t) = 0$, or $-4.9t^2 + 12.25t + 115 = 0$ is seen.	**M1**
	Valid attempt to solve quadratic equation (could be using completed square form from part **b**, calculator or formula).	**M1**
	Clearly states that $t = 6.25$ s (accept $t = 6.3$ s) is the answer, or circles that answer and crosses out the other answer, or explains that t must be positive as you cannot have a negative value for time.	**A1**
		(3 marks)
9cii	hmax = awrt 123 ft A from part b.	**B1ft**
	$t = \frac{5}{4}$ or $t = 1.25$ ft C from part b.	**B1ft**
		(2 marks)
		Total 9 marks

NOTES: c: Award 4 marks for correct final answer, with some working missing. If not correct B1 for each of A, B and C correct.

If the student answered part **b** by completing the square, award full marks for part **c**, providing their answer to their part **b** was fully correct.

10a	$\angle A = 45^\circ$ seen or implied in later working.	**B1**
	Makes an attempt to use the sine rule, for example, writing $\frac{\sin 120^\circ}{8x-3} = \frac{\sin 45^\circ}{4x-1}$	**M1**
	States or implies that $\sin 120^\circ = \frac{\sqrt{3}}{2}$ and $\sin 45^\circ = \frac{\sqrt{2}}{2}$ **NOTE:** Award ft marks for correct work following incorrect values for sin 120° and sin 45°	**A1**
	Makes an attempt to solve the equation for x. Possible steps could include: $\frac{\sqrt{3}}{16x-6} = \frac{\sqrt{2}}{8x-2}$ or $\frac{\sqrt{6}}{16x-6} = \frac{1}{4x-1}$ or $\frac{3}{16x-6} = \frac{\sqrt{6}}{8x-2}$ $(8\sqrt{3})x - 2\sqrt{3} = (16\sqrt{2})x - 6\sqrt{2}$ or $(4\sqrt{6})x - \sqrt{6} = 16x - 6$ or $24x - 6 = (16\sqrt{6})x - 6\sqrt{6}$ $6\sqrt{2} - 2\sqrt{3} = x(16\sqrt{2} - 8\sqrt{3})$ or $(4\sqrt{6})x - \sqrt{6} = 16x - 6$ or $12x - 3 = (8\sqrt{6})x - 3\sqrt{6}$	**M1ft**
	$x = \frac{6\sqrt{2} - 2\sqrt{3}}{16\sqrt{2} - 8\sqrt{3}}$ or $x = \frac{6-\sqrt{6}}{16-4\sqrt{6}}$ or $x = \frac{3\sqrt{6}-3}{8\sqrt{6}-12}$ o.e.	**A1ft**
	Makes an attempt to rationalise the denominator by multiplying top and bottom by the conjugate. Possible steps could include: $x = \frac{(3\sqrt{2}-\sqrt{3})}{(8\sqrt{2}-4\sqrt{3})} \times \frac{(8\sqrt{2}+4\sqrt{3})}{(8\sqrt{2}+4\sqrt{3})}$ $\quad x = \frac{48 + 12\sqrt{6} - 8\sqrt{6} - 12}{128 - 48}$ $\quad x = \frac{36 + 4\sqrt{6}}{80}$	**M1ft**
	States the fully correct simplifed version for x. $\quad x = \frac{9+\sqrt{6}}{20}$ *	**A1***
	NOTE: Award ft marks for correct work following incorrect values for sin 120° and sin 45°	**(7 marks)**
10b	States or implies that the formula for the area of a triangle is $\frac{1}{2}ab\sin C$ or $\frac{1}{2}ac\sin B$ or $\frac{1}{2}bc\sin A$	**M1**
	$\frac{1}{2}\left(4\left(\frac{9+\sqrt{6}}{20}\right)-1\right)\left(8\left(\frac{9+\sqrt{6}}{20}\right)-3\right)(\sin 15 \text{ or } awrt\, 0.259)$ or $\frac{1}{2}(awrt\, 1.29)(awrt\, 1.58)(\sin 15 \text{ or } awrt\, 0.259)$.	**M1**
	Finds the correct answer to 2 decimal places. 0.26	**A1**
	NOTE: Exact value of area is $\frac{1}{200}(24 + 11\sqrt{6})(\sqrt{6} - \sqrt{2})$. If 0.26 not given, award M1M1A0 if exact value seen.	**(3 marks)** **Total** **10 marks**

11a	Makes an attempt to find $\int(10-6x)\mathrm{d}x$ Raising x powers by 1 would constitute an attempt.	**M1**
	Shows a fully correct integral with limits. $\left[10x-3x^2\right]_a^{2a}=1$	**A1**
	Makes an attempt to substitute the limits into their expression. For example, $\left(10(2a)-3(2a)^2\right)-\left(10(a)-3(a)^2\right)$ or $\left(20a-12a^2\right)-\left(10a-3a^2\right)$ is seen.	**M1ft**
	Rearranges to a 3-term quadratic equation (with = 0). $9a^2-10a+1=0$	**M1ft**
	Correctly factorises the LHS: $(9a-1)(a-1)=0$ or uses a valid method for solving a quadratic equation (can be implied by correct answers).	**M1ft**
	States the two fully correct answers $a=\frac{1}{9}$ or $a=1$ For the first solution accept awrt 0.111	**A1**
		(6 marks)
11b	**Figure 1**  Straight line sloping downwards with positive x and y intercepts. Ignore portions of graph outside $0 \leqslant x \leqslant 2$	**M1**
	Fully correct sketch with points (0, 10), and ($\frac{5}{3}$, 0) labelled. Ignore portions of graph outside $0 \leqslant x \leqslant 2$	**A1**
		(2 marks)
11c	Statements to the effect that the (definite) integral will only equal the area (1) if the function is above the x-axis (between the limits) AND when $a = 1$, $2a = 2$, so part of the area will be above the x-axis and part will be below the x-axis.	**B1**
	Greater than 1.	**B1**
		(2 marks)
		Total 10 marks

12a	States that the perimeter of the track is $2\pi r + 2x = 300$ The choice of the variable x is not important, but there should be a variable other than r.	**M1**
	Correctly solves for x. Award method mark if this is seen in a subsequent step. $x = \frac{300 - 2\pi r}{2} = 150 - \pi r$	**A1**
	States that the area of the shape is $A = \pi r^2 + 2rx$	**B1**
	Attempts to simplify this by substituting their expression for x. $A = \pi r^2 + 2r(150 - \pi r)$ $A = \pi r^2 + 300r - 2\pi r^2$	**M1**
	States that the area is $A = 300r - \pi r^2$ *	**A1***
		(5 marks)
12b	Attempts to differentiate A with respect to r	**M1**
	Finds $\frac{dA}{dr} = 300 - 2\pi r$	**A1**
	Shows or implies that a maximum value will occur when $300 - 2\pi r = 0$	**M1**
	Solves the equation for r, stating $r = \frac{150}{\pi}$	**A1**
	Attempts to substitute for r in $A = 300r - \pi r^2$, for example writing $A = 300\left(\frac{150}{\pi}\right) - \pi\left(\frac{150}{\pi}\right)^2$	**M1**
	Solves for A, stating $A = \frac{22\,500}{\pi}$	**A1**
		(6 marks)
		Total 11 marks

NOTES: **12b:** Ignore any attempts at deriving second derivative and related calculations.

13a	Uses the equation of a straight line in the form $\log_4 V = mt + c$ or $\log_4 V - k = m(t - t_0)$ o.e.	**M1**
	Makes correct substitution. $\log_4 V = -\frac{1}{10}t + \log_4 40000$ o.e.	**A1**
		(2 marks)
13b	Either correctly rearranges their equation by exponentiation For example, $V = 4^{-\frac{1}{10}t + \log_4 40000}$ or takes the log of both sides of the equation $V = ab^t$. For example, $\log_4 V = \log_4(ab^t)$.	**M1**
	Completes rearrangement so that both equations are in directly comparable form $V = 40000 \times \left(4^{-\frac{1}{10}}\right)^t$ and $V = ab^t$ or $\log_4 V = -\frac{1}{10}t + \log_4 40000$ and $\log_4 V = \log_4 a + t\log_4 b$.	**M1**
	States that $a = 40\,000$	**A1**
	States that $b = 4^{-\frac{1}{10}}$	**A1**
	NOTE: 2nd M mark can be implied by correct values of a and b.	**(4 marks)**
13c	a is the initial value of the car o.e.	**B1**
	b is the annual proportional decrease in the value of the car o.e. (allow if explained in figures using their b. For example, (since b is ≈0.87) the car loses 13% of its value each year.)	**B1**
	NOTE: Accept answers that are the equivalent mathematically. For example, for b. the value of the car in 87% of the value the previous year.	**(2 marks)**
13d	Substitutes 7 into their formula from part b. Correct answer is £15 157, accept awrt £15 000	**B1ft**
		(1 mark)
13e	Uses $10000 = ab^t$ with their values of a and b or writes $\log_4 10000 = -\frac{1}{10}t + \log_4 40000$ (could be inequality).	**M1**
	Solves to find $t = 10$ years.	**A1ft**
		(2 marks)

13f	Acceptable answers include. The model is not necessarily valid for larger values of t. Value of the car is not necessarily just related to age. Mileage (or other factors) will affect the value of the car.	**B1**
		(1 mark)
		Total 12 marks